INDEX

Clausius Claperyon eqn.

$$\boxed{\frac{dP}{dT} = \frac{\Delta H}{T\Delta V}}$$

or

$$\boxed{\frac{d\ln P}{dT} = \frac{\Delta H_{vap}}{RT^2}}$$

NAME: K Sreeman Reddy STD.: ______ SEC.: ______ ROLL NO.: 2017118 SUB.: Maths

Dividend = Divisor × Quotient + Remainder

factor = Divisor which has R=0

S. No.	Date	Title	Page No.	Teacher's Sign / Remarks

Divisors of 30 = 2, 3, 10 --

Multiples of 30 = 30, 60 ---

If $\frac{p}{q}$ is the root of $a_0x^n + a_1x^{n-1} \dots a_n = 0$ then p is a factor of a_n & q is a factor a_0

$$a^3+b^3=(a+b)(a^2+b^2-ab)$$

$$n\left(\bigcup_{i=1}^{n} A_i\right) = \sum n(A_i) - \sum n(A_i \cap A_j) + \sum n(A_i \cap A_j \cap A_k) \dots$$

Power set = the set containing all subsets of A including null set

Maths

1) Algebra 2) Calculus 3) Geometry 4) Trignometry

Calculus

1) Functions 2) Limits, continuity and differentian 3) Applications of Differentiation 4) Indefinite integration 5) Definite integration 6) Area 7) Differential Eqns

Algebra

1) Sets, relations & mathematical reasoning 2) Complex numbers 3) Theory of eqns 4) Sequences & series 5) permutations and combinations 6) Probability 7) Binomial theorem 8) Matrices and determinants 9) Mathematical Induction

Geometry

1) Line & Pair of lines 2) Circles 3) Parabola 4) Ellipse 5) Hyperbola 6) Vectors 7) 3D Geometry

Trigonometry

1) Ratios & identities 2) Eqns 3) Inverse circular functions 4) Properties of Δle's 5) Heights & distances.

[D.C]

1) Functions

Set: The collection of well defined objects is called set & those objects are called elements of the set.

→ Cardinal number of infinite sets is not defined.

1) $A \cup A^c = \phi$ 2) $B - A = B \cap \bar{A} = B - A \cap B$ 3) $A \Delta B = (A-B) \cup (B-A)$
$= A \cup B - A \cap B$

4) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ 5) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

6) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

7) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Relation: Any subset of cardinal product of A & B is called a relation from A to B. $A \times B = \{a, b / \forall a \in A, b \in B\}$

1.) Reflexive relation ⇒ (x, x) always present in it if $x \in D$ where $D \in A$

2) Symmetric relation ⇒ If (a, b) is there then (b, a) must be there.

3) Antisymmetric relation ⇒ if (x, y) & (y, x) are there then $x = y$

4) Transitive relation ⇒ (x, y) & (y, z) ⇒ x, z

⇒ Domain of R = $\{x : x \in A, (x, y) \in R$ for some $y \in B\}$

$\boxed{\text{Equivalence relation} = R \,\&\, S \,\&\, T}$

Function: A relation is said to be a function if it satisfies 2 conditions

1) Every element in the domain should have an image in B.

2) Every element should have only one image.

→ We consider only real functions. So, Domain is chosen accordingly.

$f : A \to B \Rightarrow (a, b) \Rightarrow$ b = image
a = pre image or argument

$f(A)$ = Range = The set of all possible value of $f(x)$

→ no. of functions from $A \to B$ is $(n(B))^{n(A)}$

→ domain cannot be ϕ

→ a^x is defined for only $a>0$

1) Polynomial function

$f(x) = a_n x^n + a_{n-1} x^{n-1} \dots + a_0$, n = degree

→ alway continous

→ n = odd ⇒ atleast one root & Range = R

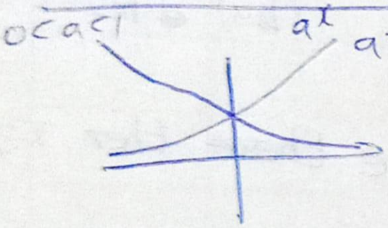
$f(x) + f\left(\frac{1}{x}\right) = f(x) f\left(\frac{1}{x}\right)$ (polynomial)

⇒ $f(x) = 1 \pm x^n$

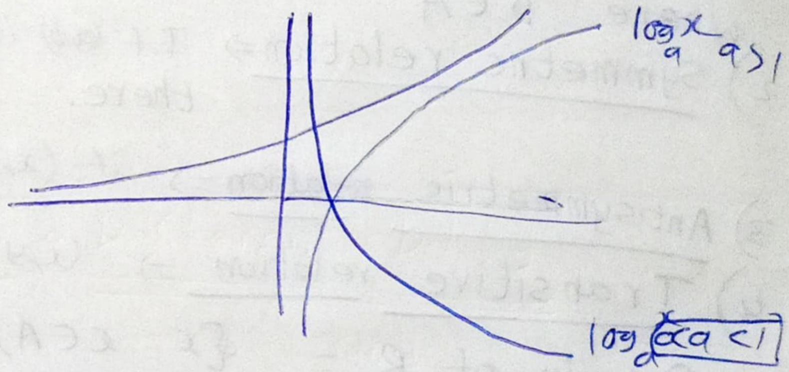
2) Algebraic function

Polynomial functions made using operators like $+, -, \div, \times, \sqrt{}$ e.tc

3) Exponential functions



4) Logarithmic functions



5) Signum function

$sgn(x) = \begin{cases} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{cases}$

6) G.I.F = $[x]$

$[x+y] \geq [x]+[y]$

$[x] + [-x] = 0 \quad x \in I$

$= -1 \quad x \notin I$

7) L.I.F = $\lceil x \rceil$

$(x) = [x]+1 \quad x \notin I$

$(x) = [x] \quad x \in I$

→ Constant function ⇒ ~~order~~ degree = 0 but

$f(x) = 0$ ⇒ degree not defined

→ $f(x) = \underbrace{\frac{f(x)+f(-x)}{2}}_{\text{even}} + \underbrace{\frac{f(x)-f(-x)}{2}}_{\text{odd}}$ (uniqely)

Equality

$f(x) = g(x) \Rightarrow D_f = D_g$, $f(x_1) = g(x_1)$, $R_f = R_g$

$\frac{1}{x} = \frac{x}{x^2}$ but $2\log x \neq \log x^2$

→ If a, b are +ve real nos and $x \in (0, \frac{\pi}{2})$ then the range of $a\sec x + b\operatorname{cosec} x$ is $\left[(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}}, \infty\right)$

→ Homogegeneous function → $f(kx) = k^n f(x)$, n = degree

⇒ Implicit → y cannot be expressed in x, Explicit → y can be expressed

⇒ One-One = Injection, Onto = surjection, $\boxed{\text{Bijection} = I+S}$

no. of f = ${}^bP_a$, no. of f = $b^a - {}^bC_1 (b-1)^a + {}^bC_2 (b-2)^a \ldots$

Bijection: $f: A \to B$ $n(A) = n(B)$ (if they are defined)

$f^{-1}(x): B \to A$

→ If $y = f(x)$ & $y = f^{-1}(x)$ intersect then they intersect either on $y = x$ or any line ⊥ to $y = x$.

Ex: $y = -x$, $xy = c^2$

⇒ If gof(x) is bijection then f must be one-one & g must be onto.

⇒ $f(x) = ax + \sin x$ is bijection if $a \in \boxed{(-\infty, -1]} \cup [1, \infty)$

⇒ symmetric about origin ⇒ $f(-x, -y) = f(x, y)$

⇒ f(x) be a real function S.T

$af(x+2) + bf(x+1) + cf(x) = 0 \Rightarrow f(x) = k_1\alpha^x + k_2\beta^x$, $\alpha, \beta \to ax^2 + bx + c$

$(k_1 + k_2x)\alpha^x$ $= \cancel{(k_1 + k_2x)^x}$ $\alpha, \alpha \to ax^2 + bx + c$

→ If $f(a-x) = -f(a+x)$ then it is symmetric about a.

Limit:

Limits

Let $f: A \to R$

Limit Definition: Given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - l| < \varepsilon$ whenever $x \in E$ and $0 < |x - a| < \delta$

In this case $\lim_{x \to a} f(x) = l$ or $f(x) \to l$ as $x \to a$

1) $\underset{x\to 0}{Lt} \frac{\sin x}{x} = 1$ 2) $\underset{x\to 0}{Lt} \frac{\tan x}{x} = 1$ 3) $\underset{x\to a}{Lt} \frac{x^n - a^n}{x-a} = na^{n-1}$ 4) $\underset{x\to 0}{Lt} \frac{\sin^{-1} x}{x} = 1$

5) $\underset{x\to 0}{Lt} \frac{\tan^{-1} x}{x} = 1$ 6) $\underset{x\to 0}{Lt} \frac{e^x - 1}{x} = 1$ 7) $\underset{x\to 0}{Lt} \frac{\ln(1+x)}{x} = 1$ 8) $\underset{x\to 0}{Lt} \frac{1-\cos^n(ax)}{x^2} = \frac{na^2}{2}$

→ L'Hospital Rule : If $f(x)$ & $g(x)$ are two differentiable functions such that $\underset{x\to a}{Lt} \frac{f(x)}{g(x)}$ takes either $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form then

$$\underset{x\to a}{Lt} \frac{f(x)}{g(x)} = \underset{x\to a}{Lt} \frac{f'(x)}{g'(x)} \text{ (untill it comes finite)}$$

Taylor's series ⇒ $\boxed{f(x) = f(0) + \frac{f'(0)x}{\lfloor 1} + \frac{f''(0)x^2}{\lfloor 2} + \cdots}$

1) $\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \cdots$ 2) $\cos x = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \cdots$

3) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 \cdots$ 4) $e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots$

5) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots$

Continuity & Differentiability

Continuity : If $\underset{x\to a}{Lt}$ exists finitely and if it is equal to $f(a)$ then $f(x)$ is said to be continous at $x=a$.

i.e $\underset{x\to a^+}{Lt} f(x) = \underset{x\to a^-}{Lt} f(x) = f(a)$

⇒ We talk about the continuity only if function is defined in the neighbour hood of a and necessarily not at a.

Removable disk : $\underset{x\to a}{Lt} f(x) \neq f(a)$ both are unequal or $f(a)$ is not defined

Missing point ~~of~~ disk : f is not defined at $x=a$

Isolated point disk : $\underset{x \to a}{Lt} f(x) \neq f(a)$

Irremovable discontinuity

1) Finite type : $\underset{x \to a^+}{Lt} f(x) \neq \underset{x \to a^-}{Lt} f(x)$ both exist finitely.

2) Infinite type : If atleast one of the ~~fo~~ R-H-L or L-H-L is not finite.

→ If $f(x)$ is disk & $g(x)$ is cont at a the $f(x)g(x)$ may be cont

Ex : $g(x) = \begin{cases} \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$, $f(x) = x$

→ If $f(x)$ & $g(x)$ both are disk the $f(x)g(x)$ & $g(x)+f(x)$ may be continous.

Eg :- $f(x) = \begin{cases} -1, & x \geq 0 \\ 1 & x < 0 \end{cases}$ $g(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$

Intermediate Value Theorem : If f is continous in $[a,b]$ & $f(a) \neq f(b)$ then for any $K \in (f(a), f(b))$ there exists atleast one c s.t $f(c) = K$

Extreme Value Theorem : If a function f is continous in $[a,b]$ then f has a max M & a min m.

→ If Domain is closed interval for a continous function then → Range also closed interval.

Differentiability : If $y = f(x)$ be a function defined in nbd of $x = a$ and it is continous at $x = a$ such that $\underset{x \to a}{Lt} \frac{f(x) - f(a)}{x - a}$ exist finitely, then the value of the limit is called derivative of the function $y = f(x)$ at it is denoted by $\frac{dy}{dx}$ or Dy or $f'(x)$

$$f'(a) = \underset{x \to a}{Lt} \frac{f(x) - f(a)}{x - a} = \underset{h \to 0}{Lt} \frac{f(a+h) - f(a)}{h}$$

$$= \underset{h \to 0}{Lt} \frac{f(a-h) - f(a)}{-h}$$

(remember '$-h$')

→ Every differentiable function is always continous but converse is not true.

⇒ If $f(x) = \begin{cases} x^n \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$

1) $n > 0$ → continous
2) $n > 1$ → differentiable but derivative not continous
3) $n > 2$ $f'(x)$ is continous but not differentiable

⇒ If the polynomial $a_0x^n + a_1x^{n-1} + \cdots + a_n = 0$, where $a_0, a_1 \cdots a_n$ are rational numbers has a rational root $\frac{p}{q}$, then p is a divisor of a_n, q is divisor of a_0

→ At $x=0$ in $f(x) = x^{\frac{1}{3}}$, tangent exists but not differentiable.

Differentiation

1) $(x^n)' = nx^{n-1}$
2) $(\sin x)' = \cos x$
3) $(\cos x)' = -\sin x$
4) $(\tan x)' = \sec^2 x$
5) $(\sec x)' = \sec x \tan x$
6) $(\operatorname{cosec} x)' = -\operatorname{cosec} x \cot x$
7) $(\cot x)' = -\operatorname{cosec}^2 x$
8) $(e^x)' = e^x$
9) $\frac{d}{dx}(\log x) = \frac{1}{x}$
10) $(a^x)' = a^x \log a$
11) $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$
12) $(\cos^{-1} x)' = \frac{-1}{\sqrt{1-x^2}}$

⇒ I The range of

$$f(x) = \underset{n \to \infty}{Lt} \left(\frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \cdots \text{n terms} \right) \text{ is}$$

$\{1, 0\}$ 0 at $\boxed{x = 0.00}$

13) $(\sec^{-1}x)' = \frac{1}{|x|\sqrt{x^2-1}}$ 14) $(\operatorname{cosec}^{-1}x)' = \frac{-1}{|x|\sqrt{x^2-1}}$

$F'(x) = G'(x) \not\Rightarrow F(x) - G(x) = c$ (true if they both are continuous and in a single Interval)

$F(x) = \begin{cases} x & x>0 \\ -x & x<0 \end{cases}$

$G(x) = \begin{cases} x+5 & x>0 \\ -x+10 & x<0 \end{cases}$

Indefinite Integration

⇒ **Integration** ($\int$) : it is the antiderivative of a function.

$$\frac{d}{dx}(f(x)) = g(x) \Rightarrow f(x) = \int g(x)\,dx + \underbrace{C}$$

constant of integration

Integral = primitive = antiderivative.

⇒ Non-integrable = antiderivative cannot be expressed in terms of elementary functions.

Eg: $\int \frac{\sin x\,dx}{x}$, $\int \frac{\cos x\,dx}{x}$, $\int \frac{dx}{\ln x}$

1) $\int \sec^2 x\,dx = \tan x$

2) $\int \operatorname{cosec}^2 x\,dx = -\cot x$

3) $\int \sec x \tan x\,dx = \sec x$

4) $\int \operatorname{cosec} x \cot x\,dx$ $- \operatorname{cosec} x$

5) $\int \sinh x = \cosh x$

6) $\int \cosh x = \sinh x$

7) $\int \operatorname{sech}^2 x = \tanh x$

8) $\int \operatorname{cosech}^2 x = -\coth x$

9) $\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x$

10) $\int \operatorname{cosech} x \coth x = -\operatorname{cosech} x$

11) $\int \frac{dx}{1-x^2} = \begin{cases} \tanh^{-1}x + c & |x|<1 \\ \coth^{-1}x + c & |x|>1 \end{cases}$

12) $\int \tan x\, dx = \int \frac{\sec x \tan x\, dx}{\sec x} = \ln(|\sec x|)$ 13) $\int \cot x\, dx = \ln|\sin x|$

14) $\int \frac{dx}{x} = \ln|x|$ (| | is necessary)

15) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$ 16) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

17) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)$ ~~18) $\int \operatorname{cosec} x\, dx = \log$~~

18) $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| = \log\left|\tan\frac{x}{2}\right|$

19) $\int \sec x\, dx = \log|\sec x + \tan x| = \log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|$

20) $\int \frac{dx}{\sqrt{x^2+1}} = \sinh^{-1} x$ 21) $\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x$

22) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|$ 23) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|$

24) $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$

25) $\int \sqrt{x^2 \pm a^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + c$

26) $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

27) $\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

28) $\int e^{ax}\cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(\cancel{a\sin bx}\ a\cos bx + b\sin bx)$

Note:

$\Rightarrow \int e^x (f(x) + f'(x)) dx = e^x f(x)$

$\Rightarrow \int e^{g(x)} (f(x) g'(x) + f'(x)) dx = e^{g(x)} f(x)$

$$\int uv\,dx = uV_1 - \int u' V_1 dx = uV_1 - u'V_2 + \int u'' V_2 dx$$
$$= uV_1 - u'V_2 + u''V_3 - \int u''' V_3 dx$$
$$= uV_1 - u'V_2 + u''V_3 - u'''V_4 \cdots$$

$\Rightarrow \int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}} \cancel{\int \frac{dt}{\sqrt{-t^2-b^2}} - \frac{1}{\sqrt{a}}} \sin^{-1} ($

$= \frac{1}{\sqrt{a}} \int \frac{d(x+c)}{\sqrt{k^2-(x+c)^2}} = \frac{1}{\sqrt{a}} \sin^{-1}\left(\frac{x+c}{k}\right) \boxed{a<0}$

$= \frac{1}{\sqrt{a}} \int \frac{d(x+c)}{\sqrt{(x+c)^2-k^2}} = \frac{1}{\sqrt{a}} \times \cancel{\frac{1}{2k}} \ln \left|(x+c) + \sqrt{(x+c)^2-k^2}\right|$

$\Rightarrow \int \frac{dx}{ax^2+bx+c} = \int \frac{dx}{a(x+c)^2+k^2} \rightarrow \tan^{-1}(g(x))$ form

$\int \frac{dx}{a(x+c)^2-k^2} \rightarrow \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right|$ form

$\Rightarrow \int \frac{dx}{a+b\sin^2 x}, \int \frac{dx}{(a\cos x + b\sin x)^2}, \int \frac{dx}{a+b\sin^2 x + c\cos^2 x} \Rightarrow \int \frac{d(\tan x)}{f(\tan x)}$

$\Rightarrow \int \frac{dx}{a\sin x + b\cos x + c}$ $\rightarrow c=0$ compound angle

$\rightarrow c \neq 0$ $\sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}$

$\Rightarrow \int \frac{p\sin x + q\cos x + r}{a\sin x + b\cos x + c} dx = \int \left(\frac{A f(x) + B f'(x) + C}{f(x)}\right) dx$

$$\int_0^{\pi/2} \sin^n x \cdot \cos^m x = \frac{[(n-1)(n-3)(n-5)\ldots 1 \text{ or } 2][(m-1)(m-3)\ldots(1 \text{ or } 2)] \cdot K}{(m+n)(m+n-2)(m+n-4)\ldots 1 \text{ or } 2}$$

$K = \frac{\pi}{2}$ if m, n both are even

$K = 1$ otherwise.

$\Rightarrow$

$$\int \frac{\phi(x)\,dx}{P(x)\sqrt{Q(x)}}\,dx \qquad \phi(x), P(x), \& \; Q(x) \text{ are polynomials}$$

Form

$Q(x)$ is linear $\qquad Q(x) = t^2$

$P(x)$ & $Q(x)$ are both pure quadratic $\qquad x = \frac{1}{t}$

i.e ax^2+b

$P(x)$ is linear, $Q(x)$ is quadratic $\qquad P(x) = \frac{1}{t}$

$P(x) = (x-k)^r$, $Q(x)$ is quadratic $\qquad P(x) = \frac{1}{t}$

Note:- Partial fractions is also very important

$\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2}+c$

$\int \cos^{-1}x\,dx = x\cos^{-1}x - \sqrt{1-x^2}+c$

$\int \tan^{-1}x\,dx = x\tan^{-1}x - \log\sqrt{1+x^2}+c$

$\int \cot^{-1}x\,dx = x\cot^{-1}x + \log\sqrt{1+x^2}+c$

$\int \sec^{-1}x\,dx = x\sec^{-1}x - \log(x+\sqrt{x^2-1})+c$

$\int \operatorname{cosec}^{-1}x\,dx = x\operatorname{cosec}^{-1}x + \log(x+\sqrt{x^2-1})+c$

Definite integration

1) $$\int_a^b f(x)\,dx = \lim_{\substack{n\to\infty\\ h\to 0}} \sum_{r=1}^{n} h f(a+rh) = \lim_{\substack{n\to\infty\\ h\to 0}} \sum_{r=0}^{n-1} h f(a+rh)$$

$$\boxed{nh = b-a}$$

2) $$\lim_{n\to\infty} \frac{1}{n} \sum_{r=\alpha(n)}^{r=\beta(n)} f\left(\frac{r}{n}\right) = \int_{\lim_{n\to\infty}\frac{\alpha(n)}{n}}^{\lim_{n\to\infty}\frac{\beta(n)}{n}} f(x)\,dx$$

Leiibnitz rule

$\Rightarrow$ $F(x) = \int_{a(x)}^{b(x)} f(x,t)\,dt$

$$F'(x) = \left(f(x, b(x))\, b'(x) - f(x, a(x))\, a'(x) \right) + \int_{a(x)}^{b(x)} \frac{\partial f(x,t)}{\partial x}\, dt$$

$\Rightarrow$ 1) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \infty = \log 2$

2) $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + - \quad \infty = \frac{\pi^2}{12}$

3) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \quad \infty = \frac{\pi^2}{6}$

4) $1 + \frac{1}{3^2} + \frac{1}{5^2} + \quad - \quad = \frac{\pi^2}{8}$

$\Rightarrow$ $\int_0^{\frac{\pi}{2}} \log(\sin x)\, dx = \int_0^{\frac{\pi}{2}} \log(\cos x)\, dx = -\frac{\pi}{2} \log 2$

$\Rightarrow$ $\int_{f(a)}^{f(b)} f^{-1}(x)\, dx + \int_a^b f(x)\, dx = b f(b) - a(f(a))$

or

$\int_a^b x f'(x) dx + \int_a^b f(x) dx = b f(b) - a f(a)$

$\Rightarrow$ $I_n = \int \sec^n x\, dx$ ($\sec^2 x$) $\boxed{V}$ $\qquad I_n = \int \cos^n x\, dx$ ($\cos x$) $\boxed{V}$

$I_n = \int \sin^m x \cos^n x\, dx$ ($\cos x$) $\qquad I_n = \int \tan^n x\, dx$ ($\tan^2 x$)

$I_n = \int \sin^m x\, dx$ ($\sin x$)

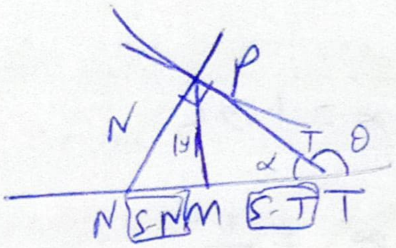
$I_n = \int_0^\infty \frac{\ln(t)\, dt}{(t+a)(t+b)}$ $\Rightarrow$ let $\boxed{t = \frac{ab}{x}}$

A.O.D

$\Rightarrow$ $\delta y = f(x+\delta x) - f(x)$
$\cong f'(x)\, dx$

relative error $= \frac{\delta y}{y}$ % err $= \frac{\delta y}{y} \times 100$

$\Rightarrow$



$S.T = \frac{|y|}{m}$

$S.N = |y| m$

$PN = |y| \sqrt{1+m^2}$

$PT = \frac{|y|}{m} \sqrt{1+m^2}$

$$\boxed{\frac{S.T}{S.N} = \left(\frac{PT}{PN}\right)^2}$$

$\Rightarrow$ The shortest distance and longest distance are along common normal

$\Rightarrow$ At $x=0$ for $f(x)=x^3$ it is increasing
At $x=0$ for $f(x)=x^{\frac{1}{3}}$ $f'(x)$ does not exist but it has tangent.

Critical points: If $f(x)$ be a function defined in I. Then the points at which $f'(x)=0$ or does not exist are critical points. End points are not critical points.

Stationary points: $f'(x)=0$

Turning point $f'(x)$ changes sign or it has extremum.

Inflection point: It should have a tangent and it double derivative exists it is equal to 0. $f''(x)$ changes sign. Ex: at $x=0$ for $x^{\frac{1}{3}}$

Rolle's theorem

In $[a,b]$ $f(x)$ is continous & Differenticable in (a,b)

$f(a)=f(b)$ $\Rightarrow$ $f'(c)=0$ for some $c \in (a,b)$

Lagrange's M.V.T

C in $[a,b]$ & Diff in (a,b)

$$\Rightarrow f'(c) = \frac{f(b)-f(a)}{b-a}$$

$g(x) = f(x) - f(a) - \left(\frac{f(b)-f(a)}{b-a}\right)(x-a)$

Note:

$$\frac{f(a+h)-f(a)}{h} = f'(a+h\theta) \quad , \theta \in (0,1)$$

Cauchey's mean value theorem

$$\phi(x) = f(x) - f(a) - \frac{(f(b)-f(a))}{g(b)-g(a)}(g(x)-g(a))$$

$$\Rightarrow \boxed{\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}}$$

Jenson's inequality

$f''(x) \geq 0$

$$f\left(\frac{x_1+x_2+x_3+x \cdots +x_n}{n}\right) \leq \frac{\Sigma f(x_i)}{n}$$

$f''(x) \leq 0$

$$f\left(\frac{x_1+x_2 \cdots x_n}{n}\right) \geq \frac{\Sigma f(x_i)}{n}$$

$\Rightarrow$ $\frac{x^2}{a} + \frac{y^2}{b} = 1$ & $\frac{x^2}{c} + \frac{y^2}{d} = 1$ cut orthagonally it $a-b = c-d$ & they have atlest one point of intersection

⇒ If for a ~~increasing~~ monotonic function $f'(x) = a + 3\sin x$ ...

⇒ $a > 5$ or $a < -5$ → don't forget this

⇒ $$f(x) = f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2!} + \dots + f^{n-1}(a)\frac{(x-a)^{n-1}}{(n-1)!} + f^n(\varepsilon)\frac{(x-a)^n}{n!}$$ where $\varepsilon = a + \theta(x-a)$, $0 < \theta < 1$

is called TAYLOR'S FORMULA. For $a = 0$ it is MACLAURIN's formula

⇒ $$f(a+h) = f(a) + \frac{h}{1!}f'(a) + \frac{h^2}{2!}f''(a+h\theta)$$

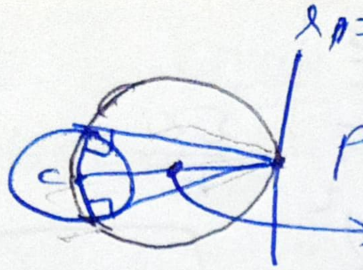
or

$$= f(a) + \frac{h}{1!}f'(a) + \frac{h^2}{2!}f''(a) + \frac{h^3}{3!}f'''(a+h\theta)$$

From this property it follows that the point & its intersection with directrix, both joined subtends right angle at the focus. ←

Note

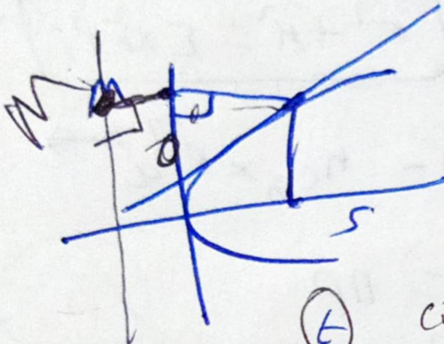
→



→ circucentre.

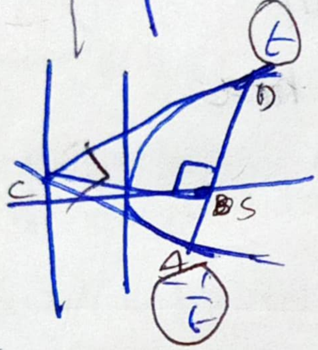
locus = line ~~[illegible]~~ ‖lel to l_1 & passing through midpoint.

→



M & S are mirror images.

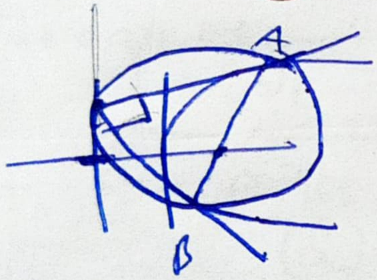
→



Circles drawn with AC and CD as diameter intersect at focus.

Focus is the ⊥lar drawn from foot of the the point of intersection of the focal chord.

→



Circle drawn as a the diameter of a focal chord always touches the directrix.

→ Circum circle of the △le formed by any 3 tangents to the parabola always passes through the focus.

→ Simson line: There exists a point P from which the projections drawn to the sides of △ABC are collinear then p always lies on the circumcircle of △ABC & the line of collinearity is called the simson line or ~~[illegible]~~ Pedal line.

→ It is true for any arbitrary point

Random Variables

μ = mean = expectation = $\frac{\Sigma x_i}{n}$

Variance = $\sigma^2 = \frac{\Sigma (x_i - \mu)^2}{n}$

σ = standard deviation

$$\boxed{\sigma^2 + \mu^2 = \frac{\Sigma x_i^2}{n}}$$

$$\boxed{\mu = \Sigma x_i \times P_i}$$

$$\boxed{V = \sigma^2 = \Sigma (x_i - \mu)^2 P_i}$$

$$\boxed{\sigma^2 + \mu^2 = \Sigma x_i^2 P_i}$$

Binomial probability

$(q+p)^n$

$P(x = r) = {}^nC_r \times p^r q^{n-r}$

x = no. of times pass

p = pass q = failure

$\mu = np$

$V = \sigma^2 = npq$

→ It is also known as bernoulli's distribution

Poisson's distribution

$n \to \infty$ $p \to 0$ but $np = \lambda$

$q \to 1$

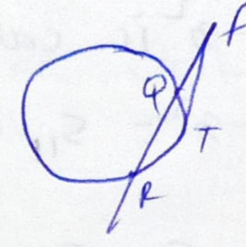
$$\boxed{V = \sigma^2 = \mu = \lambda}$$

$$P(x = r) = \frac{e^{-\lambda} \lambda^r}{\lfloor r}$$

Circle

→ $x^2+y^2+2gx+2fy+c=0 \Rightarrow C=(-g,-f) \quad r=\sqrt{g^2+f^2-c}$

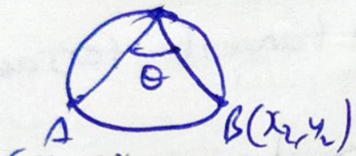
→ 

→ 

$PT^2 = PQ \times PR$

At major arc acute angle.

→ If (x_1,y_1) & (x_2,y_2) are diametrically opposite points then
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

→ 

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2) \pm \cot\theta \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0$$

→ $S+\lambda L=0$ are family passing through (x_1,y_1) & (x_2,y_2) where

$S \equiv (x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

→ The eqn of circumcircle of $\triangle ABC$ is

$\boxed{S(L)_C = L(S)_C}$ $\quad L=$ line ~~pass~~ AB

$S=$ circle with AB as diameter.

→ If four points A B C & D are concyclic

$$(S_C)(L)_D - L_C(S)_D=0$$

→ If the lines $L_1 = a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2$ meet co-ordinate axes at concyclic points.

→ ① $a_1a_2=b_1b_2$

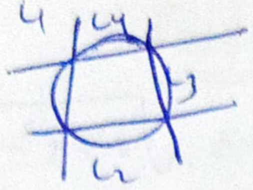
② centre $= \left(\dfrac{\text{sum of x intercept}}{2}, \dfrac{\text{sum of y-intercept}}{2}\right)$

③ Eq of circle = as all points satisfy $L_1L_2+\lambda xy=0$

⇒ we can eq co-eff of x^2 = co-eff of y^2 & co-eff $xy=0$

→ $L_1=0$, $L_2=0$ & $L_3=0$ are 3 sides of a $\triangle$le then

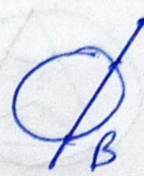
$L_1L_2+\lambda_1 L_2L_3+\lambda_2 L_3L_1$ is the circumcircle.



→ eqn $= L_1L_3 + \lambda L_2L_4 = 0$

⇒ **Power** : $cp^2 - r^2$ is called power

$cp^2 - r^2 = S_{11}$ $\quad PA$ & $PB = |S_{11}|$



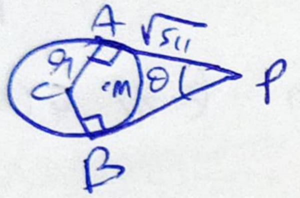
⇒ **Chord** : $S_1 + S_2 = S_{12}$ (as 1, 2 are on circle $S_{11} = 0$ & $S_{22} = 0$)

→ It passes through both 1&2

⇒ **Tangent** : $S_1 = 0$

$y = mx \pm r\sqrt{1+m^2}$ (There exists 2 tangents for same m)

⇒



$\cancel{\tan}\left(\frac{\theta}{2}\right) = \tan^{-1}\left(\frac{r}{\sqrt{S_{11}}}\right)$

⇒ Director circle : intersection of ⊥lar tangents

$\boxed{S_{11} - r^2 = 0}$

$\triangle PAB = 2 \times \frac{1}{2} \times AM \times PM$

$= \sqrt{S_{11}}\sin\frac{\theta}{2}\sqrt{S_{11}}\cos\frac{\theta}{2}$

$= \frac{S_{11}}{2} \times \frac{2 \times \frac{r}{\sqrt{S_{11}}}}{1 + \frac{r^2}{S_{11}}}$

$= \boxed{\frac{r\,(S_{11})^{3/2}}{S_{11} + r^2}}$

Chord of contact (coincides with polar if point is outside)

$S_1 = 0$

Midpoint of chord

If 1 is midpoint then

$\boxed{S_1 = S_{11}}$

→ eqn of chord is $x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = r\cos\left(\frac{\alpha-\beta}{2}\right)$

→ The locus of the foot of ⊥lar drawn from origin to a chord which subtends right angle at the origin is

Let it be h, k ⇒ eq of chord is $xh + yk = h^2 + k^2$

⇒ $x^2 + y^2 + 2gx\left(\frac{xh+yk}{h^2+k^2}\right) + 2fy\left(\frac{xh+yk}{h^2+k^2}\right) + c\left(\frac{xh+yk}{h^2+k^2}\right)^2 = 0$

⇒ Co.eff x^2 + Co.eff of $y^2 = 0$

Pole & Polar ($S_1 = 0$) (any point other than centre)

The locus of point of intersections of tangents at the ends of chord passing through pole is called polar.

1 → both are dissimilar irrational

2 → both are similar irrational (ex: $\sqrt{2}, 2\sqrt{2}$)

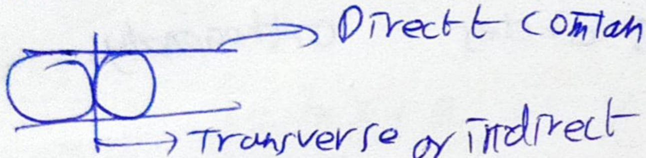
2 → 1 is rational & other is irrational

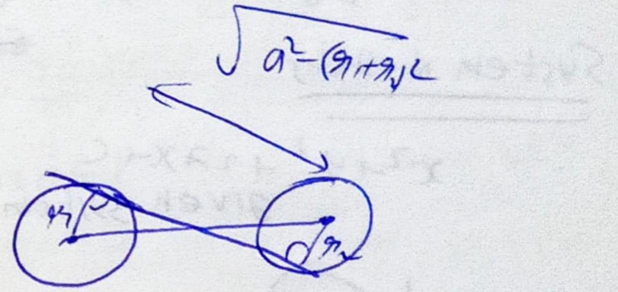
∞ → both are rational.

Conjugate points ⇒ $S_{12} = 0$

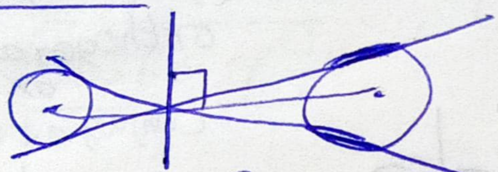
Conjugate lines $\boxed{S_1 = 0,\ S_2 = 0 \text{ if } S_{12} = 0}$

⇒ eqn of common chord is $S_1 - S_2 = 0$ where $S_1 = 0$ & $S_2 = 0$ are eqn of circles.



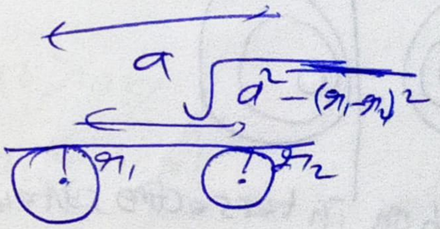


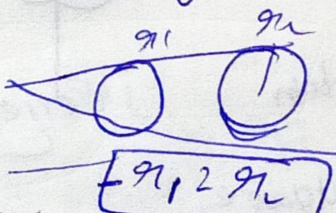
Centre of similitude



$r_1 : r_2$ ~~I-S~~ Internal C-O-S

or

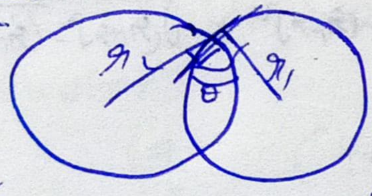


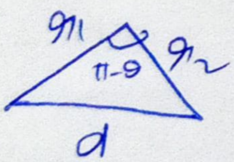


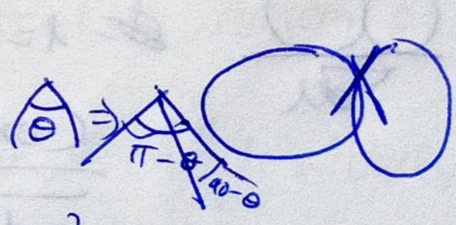
external C-O-S

$\boxed{r_1 : r_2}$

Angle between circles







$$\Rightarrow \cos\theta = \frac{d^2 - r_1^2 - r_2^2}{2 r_1 r_2}$$

$$\Rightarrow d^2 = r_1^2 + r_2^2 + 2 r_1 r_2 \cos\theta$$

⇒ Condition for orthogonality $\boxed{2(g_1 g_2 + f_1 f_2) = c_1 + c_2}$

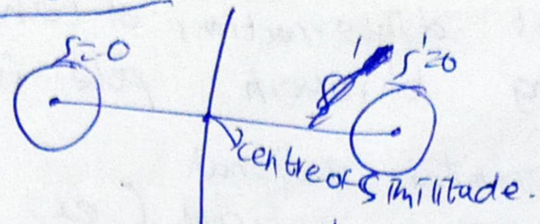
⇒ $y = mx + c$, $x^2 + y^2 = r^2$ ⇒ Point of contact $= \left(\frac{-m r^2}{c}, \frac{r^2}{c}\right)$

$c = \pm r\sqrt{1+m^2}$

Radical axis & Radical centre

Radical axis bisects the tangents

$S = S'$



⇒ It has equal powers w.r.t S & S'

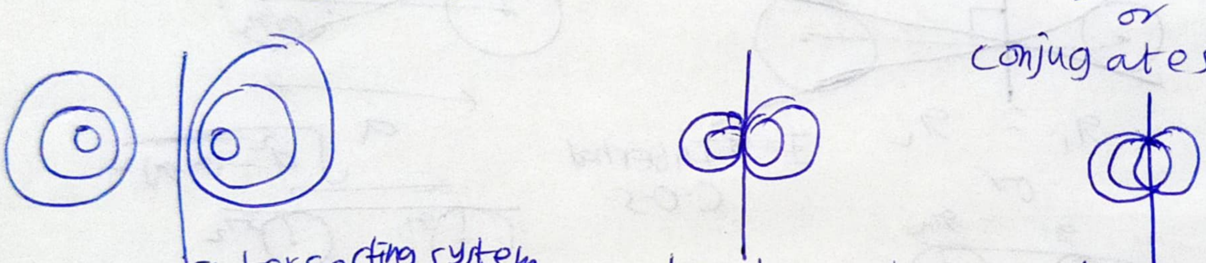
→ Radical axis of 3-circles are concurrent at a point called Radical centre

→ The circle described on R C of given 3 circles with radius as the lenghth of tangent ~~from~~ RC to any circle ~~to~~ cuts the 3 circles orthogonally.

System of circles

$x^2+y^2+2\lambda x+c$ given system

$x^2+y^2+2\mu y-c=0$ orthogonal system or conjugate system

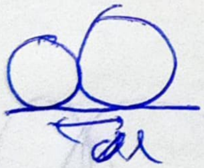


non intersecting system A

touching system B

intersecting C

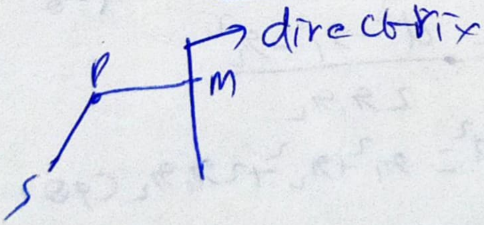
→ A, C are conjugates   B → B conjugate.



$l = \sqrt{d^2-(r_1-r_2)^2} = \sqrt{(r_1+r_2)^2-(r_1-r_2)^2} = 2\sqrt{r_1 r_2}$

Conics

definition



$\frac{SP}{PM} = e$

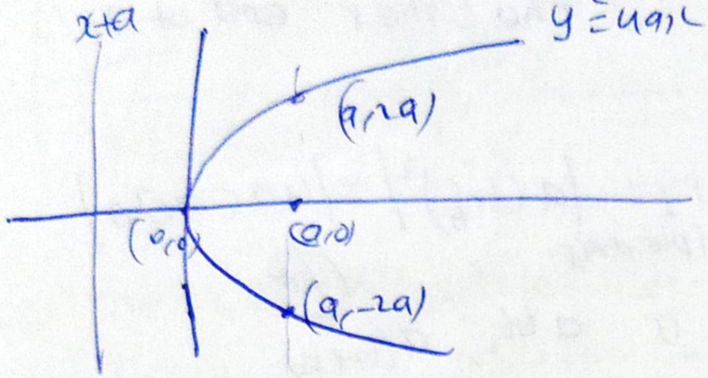
⇒ If PSQ is a focal chord of a conic

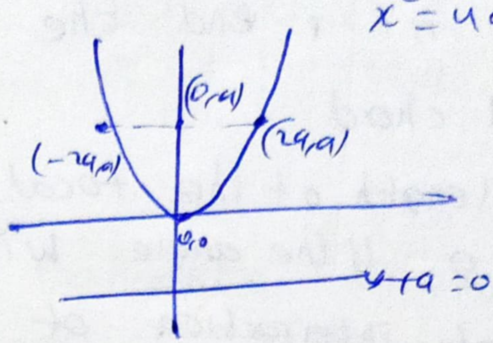
$\frac{1}{SP}+\frac{1}{SQ} = \frac{2}{l}$ $\boxed{2l = \text{length of latus rectum}}$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \qquad \Delta \neq 0$

1) $h^2 = ab \Rightarrow$ parabola 2) $h^2 - ab < 0$ ellipse

3) $h^2 - ab > 0$ hyperbola ($a+b=0 \Rightarrow$ rectangular hyperbola)

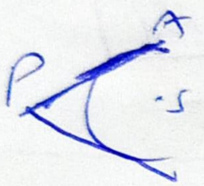




→ Tangent

$y = mx + \frac{a}{m} \qquad \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$yt = x + at^2 \qquad (at^2, 2at)$



$\boxed{SA \cdot SB = SP^2}$

→ Normal

$y + xt = 2at + at^3 \quad (at^2, 2at)$

$(am^2, -2am)$

$y = mx - 2am - am^3$

✱ $(x_1 - 2a)^3 - 27ay_1^2 > 0 \Rightarrow$ 3 distinct normals $\boxed{< 0 \Rightarrow \text{only } 1}$

$4(x_1 - 2a)^3 - 27ay_1^2 = 0 \Rightarrow$ 3 in which 2 co-incident

Diameter: The locus of the midpoints of the system of ||lel chords is called a diameter.

for $y^2 = 4ax$ $\quad y = \frac{2a}{m}$ (for the chords with slope m

→ If 3 normals are drawn from (x_1, y_1) to $y^2 = 4ax$

$G = \left(\frac{2(x_1 - 2a)}{3}, 0\right) \qquad S = \left(a + \frac{x_1}{2}, \frac{y_1}{4}\right) \qquad O \text{ or } H = \left(x_1 - 6a, \frac{-y_1}{2}\right)$

→ The line joining orthocentres of Δles formed by tangents & normals at 3 points is ||lel to the principal axis

→ $x = at^2$, $y = 2at$

→ $\frac{2}{t_1+t_2}$ is the slope of chord joining t_1, t_2

→ If t is 1 end the $\boxed{\frac{-1}{t}}$ is the other end of the focal chord

→ The length of the focal chord is $|a(t+\frac{1}{t})^2| = |4a\csc^2\theta|$ whe θ is the angle with principle axis

→ Point of intersection of t_1, t_2 is $at_1t_2,\ a(t_1+t_2)$

→ $\boxed{\tan\theta = \pm\left(\frac{\sqrt{S_{11}}}{x_1+a}\right)}$ $S_{11} = y_1^2 - 4ax_1$

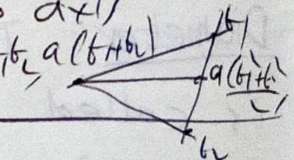
→ the eqn of pair of tangents from P_1 is $\boxed{S_1^2 = S \times S_{11}}$

→ $S_1 = S_{11}$ mid point of chord

→ The circle described ~~an~~ on a focal chord as a diameter always touches the directrix.

The line joining the point of intersection of t_1, t_2 (tangents at t_1, t_2) & their midpoint is ~~that~~ parallel to the axis

$at_1t_2,\ a(t_1+t_2)$

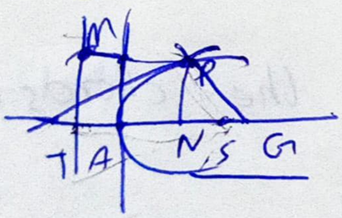


→ The area of △le formed by t_1, t_2, t_3

$a^2\,|(t_1-t_2)(t_2-t_3)(t_3-t_1)|$

→ The area of △le formed by t_1, t_2, t_3 tangent $\frac{a^2}{2}\,|(t_1-t_2)(t_2-t_3)(t_3-t_1)|$

→



$ST = SP = PM = SG$

$\Rightarrow \angle PTS = \angle TPS = \angle TPM$

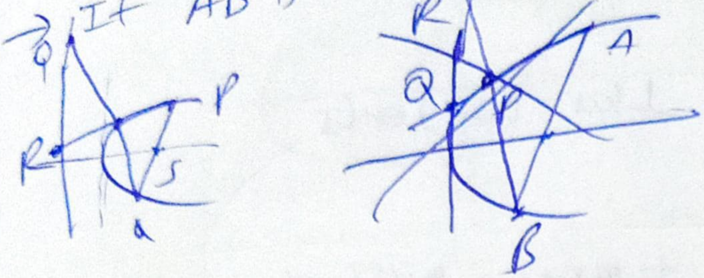
M is the reflection of S w.r.t the tangent.

→ The orthocentre of t_1, t_2, t_3 always lies on directrix.

$-a,\ a(t_1+t_2+t_3+t_1t_2t_3)$

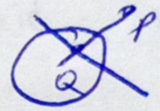
→ The tangent at the vertex is the locus of foot of ⊥lars drawn from a focus to any tangent.

→ If AB is a focal chord & P is any point on the arc AB, PA & PB are extended to meet at directrix at Q & R. QR subtend 90° at the focus.



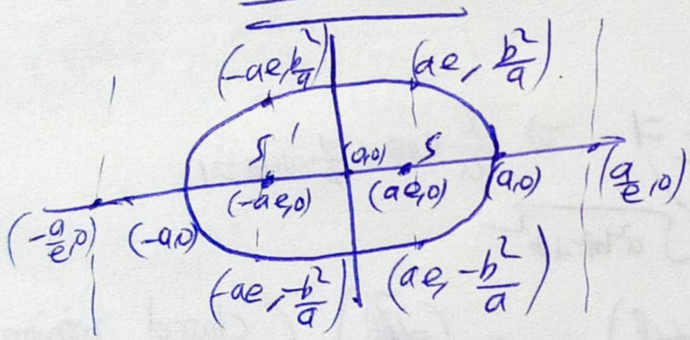
→ The circum circle of any 3 tangents will pass through the focus.

⇒ <u>Inverse points</u>: If Q is the intersection of CP & Polar of P then P & Q are inverse points.



(They are also conjugate points)

Ellipse



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\boxed{e^2 = 1-\frac{b^2}{a^2}}$$

$$SP + S'P = 2a$$

$$SP = e\left|x - \frac{a}{e}\right|$$

⇒ $y = mx + c$ is a tangent ⇒ $\boxed{c^2 = a^2m^2+b^2}$

& P.O. Tangency $= \left(-\frac{a^2m}{c}, \frac{b^2}{c}\right)$

⇒ If tangents are drawn from (x_1, y_1)

⇒ $(y_1 - mx_1)^2 = a^2m^2 + b^2$

⇒ The product of ⊥lars drawn form the focii to a tangent of the ellipse is always b^2

⇒ $x^2+y^2 = a^2$ is the auxilary circle (locus of foot of ⊥lars drawn from foci on any tangent.)

$\Rightarrow \quad \tan\theta = \dfrac{\pm 2ab\sqrt{\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1}}{x_1^2+y_1^2-a^2-b^2} = \pm\dfrac{2ab\sqrt{S_{11}}}{\text{Eq of directrix}}$

→ **Director circle** $(x^2+y^2=a^2+b^2)$;

The locus of intersection of ⊥lar tangents.

→ **Diameter :**

$$y = -\frac{b^2}{a^2 m}(x) \quad \text{for} \quad y = mx + c, \quad m \text{ is fixed}, \ \boxed{c \in R} \text{ variable.}$$

$\Rightarrow m_1 m_2 = -\dfrac{b^2}{a^2}$, where $y = m_1 x$, $y = m_2 x$ are conjugate diameters.

i-e 1 diameter bisects the chords ||let to the other.

Normal

$$\frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2$$

$$y = mx \pm \frac{m(a^2-b^2)}{\sqrt{a^2+b^2m^2}}$$

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$$

Tangent;

$$S_1 = 0 \Rightarrow \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \Rightarrow \frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

$$y = mx \pm \sqrt{a^2m^2+b^2}$$

$\Rightarrow \dfrac{x}{a}\cos\left(\dfrac{\alpha+\beta}{2}\right) + \dfrac{y}{b}\sin\left(\dfrac{\alpha+\beta}{2}\right) = \cos\left(\dfrac{\alpha-\beta}{2}\right)$ (Chord joining α, β)

→ Point of intersection of tangents at α, β is

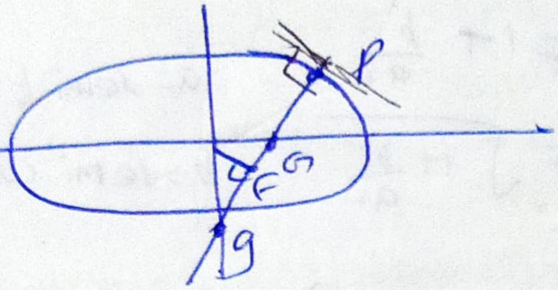
$$\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \quad \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$$

→ The area of △le formed by

$$\left|2ab\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\beta-\gamma}{2}\right)\sin\left(\frac{\gamma-\alpha}{2}\right)\right|$$

→ If $\alpha, \beta, \gamma, \delta$ are concyclic $\Rightarrow \alpha+\beta+\gamma+\delta = 2n\pi$

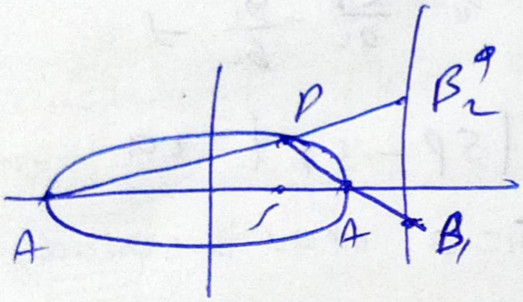
→ The sum of eccentric angle of feet of conormal points is $(2n+1)\pi$

⇒ 

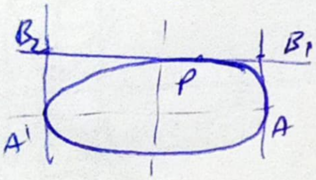
$PF \times PG = b^2$

$PF \times Pg = a^2$

$$\boxed{\frac{PG}{Pg} = \frac{b^2}{a^2}}$$

→ 

$B_1 B_2$ subtends 90° at the respective focus

→ 

$$\boxed{AB_1 \times A'B_2 = b^2}$$

→ $\frac{x^2}{6-a} + \frac{y^2}{a-2} = 1$ no. of ellipses $\neq 3$ for $a=4$ it is circle.

→ $L \approx \pi\left(3(a+b) - \sqrt{(3a+b)(3b+a)}\right)$

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circles

→ If a circle cuts two circles orthogonally then its centre passes through the radical axis of those 2 circles.

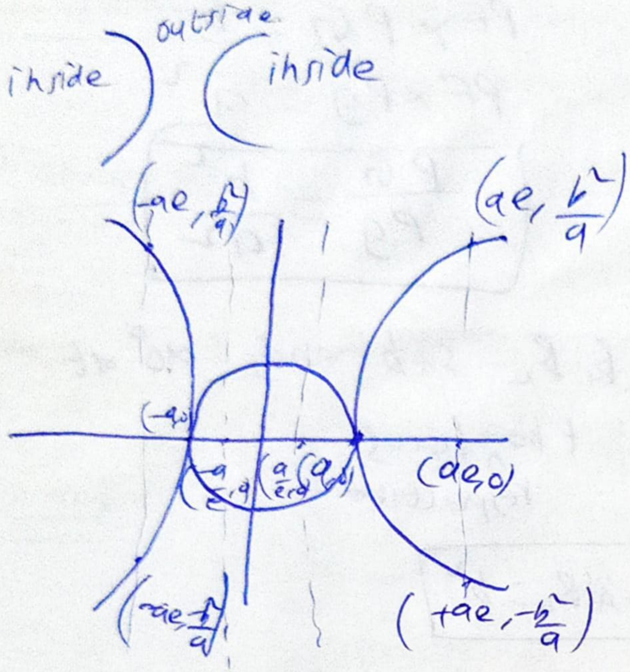
Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad e^2 = 1 + \frac{b^2}{a^2}$$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

a = semi transverse axis

b = semi conjugate axis



$$S_{11} = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$$

$$|SP - SP'| = 2a$$

$S_1 = 0$ chord of contact, tangent

Tangent:

$$\frac{x x_1}{a^2} - \frac{y y_1}{b^2} = 1 \qquad \frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

Normal:

$$\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2 \qquad \frac{ax}{\sec\theta} + \frac{by}{\sin\theta} = a^2 + b^2$$

$$\tan\theta = \pm\left(\frac{2ab\sqrt{-S_{11}}}{x_1^2 + y_1^2 - a^2 + b^2}\right) \quad \left(\boxed{-S_{11}} \Rightarrow \text{not } S_{11}\right)$$

$\Rightarrow$ $x^2 + y^2 = a^2$ (auxilary) = foot of ⊥lars from focus to any tangent.

$x^2 + y^2 = a^2 - b^2$ (Director) = locus of intersection of ⊥lar tangents.

$\Rightarrow$ chord for (α, β)

$$\frac{x}{a}\cos\left(\frac{\alpha - \beta}{2}\right) - \frac{y}{b}\sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha + \beta}{2}\right)$$

⇒ The product ⊥lars from S to any tangent is b^2

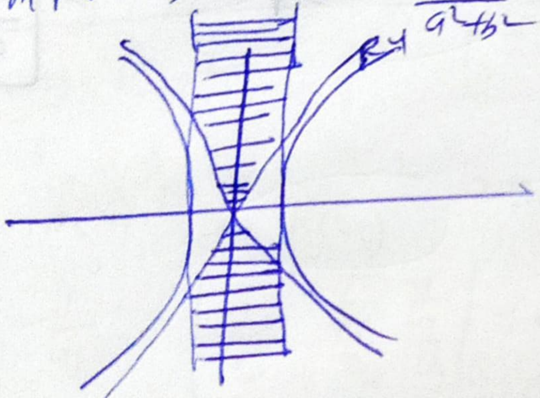
⇒ Asymptotes are not tangents

$H+C=2A$ (where H is the equation of general hyperbola & C = conjugate. A = eqn of asymptotes.

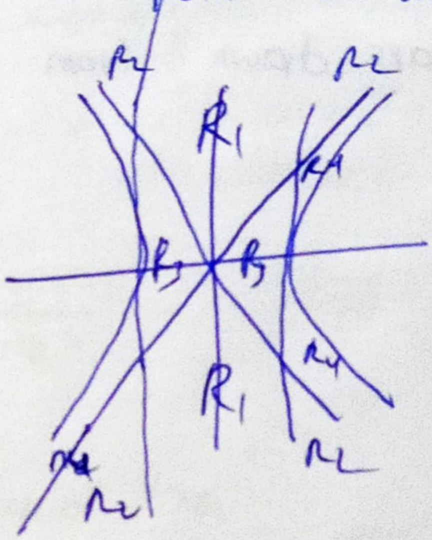
⇒ θ = angle betwn asymptotes $= 2\tan^{-1}\left(\frac{b}{a}\right) = 2\sec^{-1}(e) = \cos^{-1}\left(\frac{a^2-b^2}{a^2+b^2}\right)$.

⇒ The portion of the tangent at P formed with asymptotes is always bisected at P

⇒ The product of ⊥lar from a point on the hyperbola to its asymptotes is $\frac{a^2b^2}{a^2+b^2}$



or



R_1 → touch both on same side of transverse axis

R_2 → both opposite sides

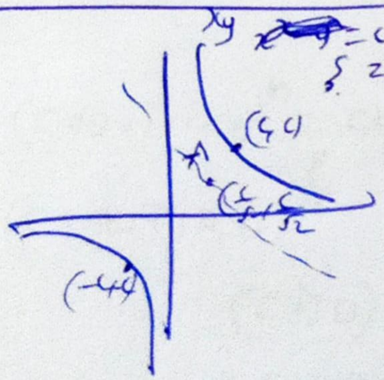
R_3 → one on opposite sides

R_4 → one same side

Rectangular hyperbola

$xy = c^2$

$S \equiv (\sqrt{2}c, \sqrt{2}c)$ $\boxed{c^2 = \frac{a^2}{2}}$



$\left(ct, \frac{c}{t}\right)$

tangent

$\boxed{x + yt^2 = 2ct}$ tangent

$\boxed{t_1 t_2} \rightarrow \left(\frac{2ct_1t_2}{t_1+t_2}, \frac{2c}{t_1+t_2}\right)$

$\boxed{\text{slope joining } t_1 t_2 \quad \frac{-1}{t_1 t_2}}$

Normal

$\boxed{xt^3 - yt = c(t^4 - 1)}$

If it passes through throught (x_1, y_1) with t_1, t_2, t_3, t_4 as roots

$\Rightarrow$ $\boxed{t_1 t_2 t_3 t_4 = -1}$

$\Rightarrow$ Orthocentre of $\left(ct_1, \frac{c}{t_1}\right), t_2, t_3$ is $\left(-\frac{c}{t_1 t_2 t_3}, -ct_1 t_2 t_3\right)$

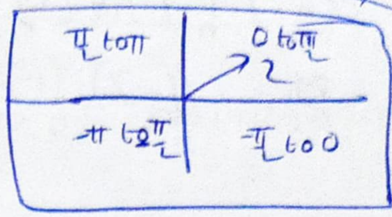
$\Rightarrow$ orthocentre of any 3 points among the feet of the normals is co-incident with the fourth point.

$\Rightarrow$ If $\left(t_1, \frac{c}{t_1}\right), t_2, t_3, t_4$ are concyclic then $\boxed{t_1 t_2 t_3 t_4 = 1}$

$\Rightarrow$ Product of $\perp$lars drawn from any point to asymptotes is $\boxed{\frac{a^2b^2}{a^2+b^2}}$

Complex Numbers

⇒ $z = a + ib = |z| e^{i\arg(z)}$ $\quad -\pi < \arg \le \pi \quad$ ~~$\arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$ is for~~



⇒ $\bar{z} = a - ib$

⇒ $z\bar{z} = |z|^2$

⇒ $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

⇒ $\overline{z_1 z_2 \cdots z_n} = \bar{z}_1 \cdot \bar{z}_2 \bar{z}_3 \cdot \bar{z}_4 \cdot \bar{z}_5 \cdots \bar{z}_n$

⇒ $\overline{z^n} = (\bar{z})^n$, $\quad z + \bar{z} = 2\,Re(z)$, $\quad z - \bar{z} = 2i\,Im(z)$

⇒ $\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$ (Euler's form)

⇒ $\arg(z_1 z_2 \cdots z_n) = \arg(z_1) + \cdots + \arg(z_n) + 2k\pi$

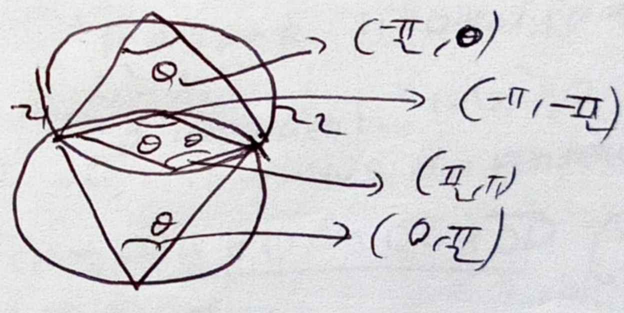
⇒ $A(z_1) \quad \lambda \quad z \quad m \quad B(z_2)$ $\qquad z = \dfrac{\lambda z_2 + m z_1}{\lambda + m}$

⇒ $-\dfrac{1}{4i} \begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix} = \text{area}$

⇒ $a\bar{z} + \bar{a}z + b = 0$ is the eqn of line

⇒ $z\bar{z} + a\bar{z} + \bar{a}z + b = 0$ is the eqn of circle.

⇒ $\arg\left(\dfrac{z - z_1}{z - z_2}\right) = \theta$



⇒ $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

⇒ $(\cos\theta + i\sin\theta)^{\frac{p}{q}}$ has q values.

$$(\text{cis}\,\theta)^{\frac{p}{q}} = \text{cis}\left((2k\pi + \theta)\frac{p}{q}\right)$$

$k = 0, 1, 2, \ldots, q-1$

⇒ $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

⇒ $z^n = 1 \Rightarrow z = \text{cis}\left(\dfrac{2k\pi}{n}\right)$, $k = 0, 1, 2, \ldots, n-1$

$\rightarrow \quad 1 + \alpha^{p} + \alpha^{2p} + \dots \alpha^{(n-1)p} = 0 \quad \text{if} \quad p \neq nk$

$= n \quad p = nk$

$\rightarrow \quad \sin\left(\frac{\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right) - \sin\left(\frac{(n-1)\pi}{n}\right) = \frac{n}{2^{n-1}}$

or $\quad (1-\alpha)(1-\alpha^2)(1-\alpha^3) - (1-\alpha^{n-1})_2 \quad \lim_{z \to 1} \left|\frac{z^n - 1}{z-1}\right|$

$\rightarrow \quad |z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\, z_1 \cdot z_2$

$z_1 \cdot z_2 = \frac{\bar{z}_1 z_2 + \bar{z}_2 z_1}{2}$

Vectors & 3D-Geometry

→ $\underset{A}{(x_1,y_1,z_1)}$ l $\dot{i}$ m $\underset{B}{(x_2,y_2,z_2)}$ $\qquad \dfrac{l(x_2,y_2,z_2)+m(x_1,y_1,z_1)}{l+m}$

→ $\left(\dfrac{\Sigma x_i}{4}, \dfrac{\Sigma y_i}{4}, \dfrac{\Sigma z_i}{4}\right)$ is the centre of tetrahedron.

→ $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ for any line.

→ angle between 2 lines $\cos\theta = \pm(l_1l_2+m_1m_2+n_1n_2)$

$\sin\theta = \sqrt{\Sigma(l_1m_2-m_1l_2)^2}$ & $\tan\theta = \pm\dfrac{\sqrt{\Sigma(l_1m_2-l_2m_1)^2}}{l_1l_2+m_1m_2+n_1n_2}$

→ Distance between the 2 lines is $\dfrac{|D|}{\sqrt{\Sigma(a_1b_2-a_2b_1)^2}}$

$$D = \begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

Vectors

→ localised vectors → have initial & final points

→ $\overline{AB} = \overline{OB}-\overline{OA}$

→ $l_1\bar a_1 + l_2\bar a_2 + \dots + l_n\bar a_n = 0$ ⇒ they are linearly dependent $\quad l_1^2+l_2^2+\dots+l_n^2\neq 0$

→ 4 or more vectors are always lineary dependent.

→ any set of vectors containing $\bar 0$ is always dependent.

→ In a Tetrahedron ABCD $\quad \overline{OG} = \dfrac{\overline{OA}+\overline{OB}+\overline{OC}+\overline{OD}}{4}$ (3:1 ratio)

→ $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}$ ⇒ eqn of line with (a, b, c) as d.r's

→ $\bar r = \bar a + s\bar b + t\bar c$ (eqn of plane) $\quad \bar r = \bar a + t\bar b$ (eqn of line)

$\bar a\cdot\bar b = |a||b|\cos(\bar a,\bar b)$

→ $(\bar a\pm\bar b)^2 = |a|^2+|b|^2\pm 2\bar a\cdot\bar b$

$|\bar a-\bar b|^2+|\bar b-\bar c|^2+|\bar c-\bar a|^2 \le 3(|a|^2+|b|^2+|c|^2)$

$\bar a - \left(\dfrac{\bar a\cdot\bar n - p}{|n|^2}\right)\times\bar n$ (Projection of $\bar a$ on the plane $\bar r\cdot\bar n = p$)

$\Rightarrow \quad \bar{r} \cdot \hat{n} = p$ (non parametric eqn of a plane)

$\Rightarrow \triangle OAB \quad$ Area $= \frac{1}{2}|\bar{a} \times \bar{b}|$

$\Rightarrow$ **Note**: If $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$ is a vector which is equally inclined with $\bar{a}, \bar{b}, \bar{c}$ then

$\bar{d} = \lambda(\hat{a} \times \hat{b} + \hat{b} \times \hat{c} + \hat{c} \times \hat{a})$ (it can be proved easily by keeping $\bar{d} = x(\hat{a} \times \hat{b}) + y(\hat{b} \times \hat{c}) + z(\hat{c} \times \hat{a})$)

$\Rightarrow$ If $\bar{a}, \bar{b}, \bar{c}$ are of equal magnitude & they are equally inclined to each other only then it is

$\lambda(\bar{a} + \bar{b} + \bar{c})$ (not always applicable).

Scalar Triple Product

$$\bar{a} \cdot (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \cdot \bar{c} = [\bar{a}\ \bar{b}\ \bar{c}] = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$[l\bar{a}\ \ m\bar{b}\ \ n\bar{c}] = lmn\,[\bar{a}\ \bar{b}\ \bar{c}]$$

$[\bar{a}\ \bar{b}\ \bar{c}] > 0 \Rightarrow$ Right handed system

Let

$$\bar{a} = a_1\bar{l} + a_2\bar{m} + a_3\bar{n}$$
$$\bar{b} = b_1\bar{l} + b_2\bar{m} + b_3\bar{n}$$
$$\bar{c} = c_1\bar{l} + c_2\bar{m} + c_3\bar{n}$$

$$[\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} [\bar{l}\ \bar{m}\ \bar{n}]$$

$$[\bar{a}\ \bar{b}\ \bar{c}](\bar{l} \times \bar{m}) = \begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \\ \bar{a} \cdot \bar{l} & \bar{b} \cdot \bar{l} & \bar{c} \cdot \bar{l} \\ \bar{a} \cdot \bar{m} & \bar{b} \cdot \bar{m} & \bar{c} \cdot \bar{m} \end{bmatrix} \star$$

(let $[\bar{a}\ \bar{b}\ \bar{c}](\bar{l} \times \bar{m}) = x\bar{a} + y\bar{b} + z\bar{c}$

$$[\bar{a}\ \bar{b}\ \bar{c}][\bar{l}\ \bar{m}\ \bar{n}] = \begin{bmatrix} \bar{a} \cdot \bar{l} & \bar{b} \cdot \bar{l} & \bar{c} \cdot \bar{l} \\ \bar{a} \cdot \bar{m} & \bar{b} \cdot \bar{m} & \bar{c} \cdot \bar{m} \\ \bar{a} \cdot \bar{n} & \bar{b} \cdot \bar{n} & \bar{c} \cdot \bar{n} \end{bmatrix} \Rightarrow [\bar{a}\ \bar{b}\ \bar{c}]^2 = \begin{bmatrix} \bar{a} \cdot \bar{a} & \bar{a} \cdot \bar{b} & \bar{a} \cdot \bar{c} \\ \bar{b} \cdot \bar{a} & \bar{b} \cdot \bar{b} & \bar{b} \cdot \bar{c} \\ \bar{c} \cdot \bar{a} & \bar{c} \cdot \bar{b} & \bar{c} \cdot \bar{c} \end{bmatrix}$$

Volume of parallelopiped with $\bar{a}, \bar{b}, \bar{c}$ as coterminal edges is

$[\bar{a}\ \bar{b}\ \bar{c}]$

Volume of tetrahedron is $\frac{1}{6}[\bar{a}\ \bar{b}\ \bar{c}]$

Integration

Gamma function

$$\boxed{\Gamma(n+1) = \int_0^\infty e^{-x} x^n \, dx = \lfloor n}$$

$$\Gamma(1) = 1$$

$$\left(\int_0^\infty x^2 e^{-x^2} dx\right) \quad (\text{Put } \boxed{x=\sqrt{t}})$$

$$\boxed{\lfloor \tfrac{1}{2} = \frac{\sqrt{\pi}}{2}} \quad \boxed{\lfloor \tfrac{-1}{2} = \sqrt{\pi}}$$

Beta

$$\beta(m+1, n+1) = \boxed{\int_0^1 x^m (1-x)^n dx = \frac{\lfloor m \; \lfloor n}{\lfloor m+n+1}} = \frac{\Gamma(m+1)\,\Gamma(n+1)}{\Gamma(m+n+2)}$$

(not $m+n$)

$$I = \int_0^1 x^m (1-x)^n dx$$

$$I_{m,n} = \int \sin^m\theta \cos^n\theta \, d\theta = \frac{n-1}{m+1} I_{m+2,\, n-2}$$

$$\sin^m\theta \cos\theta \times \cos^{n-1}\theta$$

$$I = \frac{n}{m+1} \int_0^1 x^{m+1} (1-x)^{n-1} dx$$

Binomial theorem

$$(x+a)^n = {}^nC_r \, x^{n-r} a^r$$

Numerically greatest term

$$\left[\frac{(n+1)|x|}{|x|+1}\right] = p \Rightarrow T_{p+1} \text{ is the greatest}$$

$$(\text{If } p = I \Rightarrow T_p = T_{p+1})$$

$$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$$

$$\boxed{\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}}$$

$$\boxed{C_0 + C_1 + C_2 + \cdots + C_n = 2^n}$$

$$\boxed{aC_0 + (a+d)C_1 + (a+2d)C_2 \cdots\cdots + (a+nd)C_n = (2a+nd)\,2^{n-1}}$$

$$\boxed{C_0 C_r + C_1 C_{r+1} + \cdots\cdots + C_{n-r} C_n = {}^{2n}C_{n+r} = {}^{2n}C_{n-r}}$$

$$C_0^2 + C_1^2 + C_2^2 + \cdots + C_n^2 = {}^{2n}C_n$$

$$\cancel{(1-x)}^{-n} = 1 + {}^nC_1 x + {}^{n+1}C_2 x^2 + \cdots + {}^{n+r-1}C_r x^r$$

$$(1-x)^{-n} = (1 + \cancel{x} + x^2 \cdots)^n \Rightarrow t_1 + t_2 + \cdots + t_n = r, \; t_i \in [0, \infty)$$

$$\Rightarrow \boxed{{}^{n+r-1}C_r \; \boxed{\text{ways}} = {}^{n+r-1}C_{n-1}} \quad t_i \in \mathbb{Z}$$

~~For~~ $\cancel{C_0 C_r + C_1 C_r + \cdots C_{n-r} C_n = \text{co-efficient of } x^r (C_0 + C_1 x + \cdots C_r x^r)}$

$$\cancel{\left(C_n + \frac{C_{n-1}}{x} + \frac{C_{n-2}}{x^2} \cdots\right)}$$

$$\cancel{= (1+x)^n \left(1 + \frac{1}{x}\right)^n = {}^{2n}C_{n+r}}$$

$$C_0 C_r + C_1 C_{r+1} + \cdots C_{n-r} C_n = x^r \text{ in } \left(C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \cdots \frac{C_n}{x^n}\right)\left(C_0 + C_1 x + C_2 x^2 + \cdots\right)$$

$${}^{2n}C_{n+r} = {}^{2n}C_{n-r} = C_0 C_r x^r + \frac{C_1}{x} C_{r+1} x^{r+1} \cdots C_r x^r$$

$${}^mC_0 \, {}^nC_r + {}^mC_1 \, {}^nC_{r-1} + \cdots + {}^mC_r \, {}^nC_0 = {}^{m+n}C_r$$

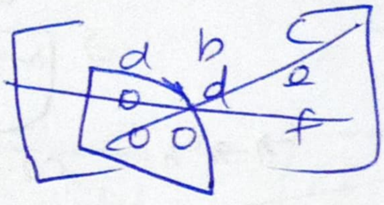
Matrices & Determinants

Matrice: Arrangement of elements in a rectangular array.

* horizontal lines = rows   vertical = coloumns.

Lower triangular matrix

$$\begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$$



Upper triangular matrix

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

$Tr(kA) = k\,Tr A$ $Tr(A+B) = TrA + TrB$ $\boxed{Tr(AB) = Tr(BA)}$

Symmetric $A^T = A$ **Skew symmetric** $A^T = -A$

Hermitian $A^\theta = A$ **Skew-Hermition** $A^\theta = -A$

$(A^\theta = (\bar{A})^T)$

Unitary $AA^\theta = I$ **Orthogonal** $AA^T = I$

Involuntary $A^2 = I$ **Idempotent** $A^2 = A$

Nilpotent $A^n = 0,\ A^{n-1} \neq 0$

$\Rightarrow n =$ index of the Nilpotent matrix.

singular matrix $\Rightarrow \boxed{|A| = 0}$

Multiplication

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} p_1 & p_2 & p_3 \\ p_4 & p_5 & p_6 \\ p_7 & p_8 & p_9 \end{bmatrix} = \begin{bmatrix} ap_1+bp_4+cp_7 & , & , \\ & , & , \\ & , & , \end{bmatrix}$$

$\Rightarrow$ If A is a non-zero matrix S.T $AB = 0$ where 0 is a null matrix. Then $|B| = 0$

Determinat: For any square matrix.

$\det(A) = a_1A_1 + b_1B_1 + c_1C_1$

$\Rightarrow A_1, B_1, C_1$ are co-factors of a_1, b_1, c_1

$a_1A_2 + b_1B_2 + c_1C_2 = 0$

$a_1B_1 + a_2B_2 + a_3B_3 = 0$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_3 & b_3 & c_3 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+ka_2 & b_1+kb_2 & c_1+kc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$|KA| = k^n|A|$ (n = order of A)

$|AB| = |A||B|$ (if both exists. $\begin{vmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{vmatrix} = acf$

__Adjoint__: The $\boxed{\text{transpose}}$ of a square matrix obtained by replacing the element with corresponding co-factors is called Adjoint of A. It is denoted by Adj (A).

$A\,\text{Adj}(A) = |A|I$

$\Rightarrow |\text{Adj}A| = |A|^{n-1}$

$\text{Adj}A\,\text{Adj}(\text{Adj}A) = |\text{adj}A|I =$

$\Rightarrow |\text{Adj}(\text{Adj}A)| = |\text{Adj}A|^{n-1} = |A|^{(n-1)^2}$

$|\text{Adj}(\text{Adj} \ldots \text{Adj}A| = |A|^{(n-1)^m}$ (m Adj times)

$\Rightarrow$ det of odd ordered skew-symmetric matrix is zero.

$(ABC)^T = C^T B^T A^T \qquad (ABC)^{-1} = C^{-1} B^{-1} A^{-1}$

$\text{Adj}(ABCD) = \text{Adj}(D)\,\text{Adj}(C)\,\text{Adj}(B)\,\text{Adj}(A)$

Sub Matrix

The matrix obtained by deleting some rows or some columns or both rows & columns is called sub-matrix of the given equation.

Rank

The highest order of non-singular sub-matrix is called rank of the given matrix.

Rank of nth order unit matrix is n

Rank of null matrix is not defined

$\text{Rank}(A') = \text{Rank}(A)$

Echelon form of a matrix

1) The first non-zero element in any row is 1 & below one only zeroes should be there.

2) The no. of zeroes before 1 in any non-zero row is less than the no. of zeroes in the next row.

$AX = B \qquad [A:B] \rightarrow R_2$

$A \rightarrow R_1$

1) $R_1 = R_2 = n \rightarrow$ Unique soln

2) $R_1 = R_2 < n \rightarrow$ infinite soln where n = no. of variables

3) $R_1 \neq R_2 \rightarrow$ no solution

Cramer's rule

consistent means atleast 1 solution
Inconsistent means no solution

$AX = B$ & $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

$x = \frac{\Delta x}{\Delta}$, $y = \frac{\Delta y}{\Delta}$ $z = \frac{\Delta z}{\Delta}$ $\Delta x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Caylay - Hamilton theorem

$A^2 - Tr(A)\, A + |A|\, I_2 = 0$ (for order 2 square matrix)

$A^3 - TrA\,(A^2) + \frac{1}{2}\left((TrA)^2 - Tr(A^2)\right)A - |A|\, I_3 = 0$ (for 3rd order square matrix)

$\Rightarrow ABA^{-1} = B^2 \Rightarrow A \cdot B^n = B^{2n} A$ & $A^n B = B^{2^n} A^n$

Trignometry

→ $\sin(\theta_1+\theta_2+\dots\theta_n) = \cos\theta_1 \cos\theta_2 \dots \cos\theta_n [S_1 - S_3 + S_5 - S_7 \dots]$

→ $\cos(\theta_1+\theta_2+\dots+\theta_n) = \cos\theta_1 \cos\theta_2 \dots \cos\theta_n [1 - S_2 + S_4 \dots]$

→ $\tan(\theta_1+\theta_2\dots+\theta_n) = \dfrac{S_1 - S_3 + S_5 - S_7 \dots}{1 - S_2 + S_4}$

$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)$

$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\cos C - \cos D = -2\sin\left(\frac{C-D}{2}\right)\sin\left(\frac{C+D}{2}\right)$

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$\boxed{\cos 3A = 4\cos^3 A - 3\cos A}$ & $\boxed{\sin 3A = 3\sin A - 4\sin^3 A}$

$\sin\theta \sin\left(\frac{\pi}{3}+\theta\right)\sin\left(\frac{\pi}{3}-\theta\right) = \frac{1}{4}\sin 3\theta$

$\cos\theta \cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right) = \frac{1}{4}\cos 3\theta$

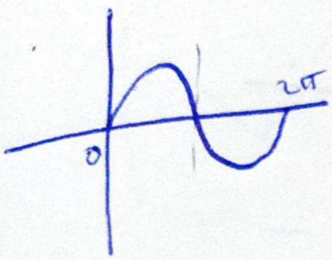
$\tan\theta \tan\left(\frac{\pi}{3}-\theta\right)\tan\left(\frac{\pi}{3}+\theta\right) = \tan 3\theta$	$\cot\theta \cot\left(\frac{\pi}{3}-\theta\right)\cot\left(\frac{\pi}{3}+\theta\right) = \cot 3\theta$
$\cos\theta + \cos\left(\frac{2\pi}{3}+\theta\right) + \cos\left(\frac{2\pi}{3}-\theta\right) = 0$	$\cos\theta - \cos\left(\theta+\frac{\pi}{3}\right) - \cos\left(\theta-\frac{\pi}{3}\right) = 0$
$\sin\theta + \sin\left(\theta+\frac{2\pi}{3}\right) + \sin\left(\theta-\frac{2\pi}{3}\right) = 0$	$\sin\theta - \sin\left(\theta+\frac{\pi}{3}\right) - \sin\left(\theta-\frac{\pi}{3}\right) = 0$

$\sin\alpha + \sin(\alpha+\beta) + \dots + \sin(\alpha+(n-1)\beta) = \dfrac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\alpha+\frac{(n-1)\beta}{2}\right)$

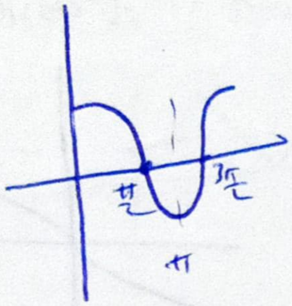
$\cos\alpha + \cos(\alpha+\beta) + \dots + \cos(\alpha+(n-1)\beta) = \dfrac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\alpha+\frac{(n-1)\beta}{2}\right)$

$\cot\theta + \tan\theta = 2\operatorname{cosec}2\theta$	$\cot\theta - \tan\theta = 2\cot 2\theta$
$\operatorname{cosec}2\theta + \cot 2\theta = \cot\theta$	$\operatorname{cosec}2\theta - \cot 2\theta = \tan\theta$

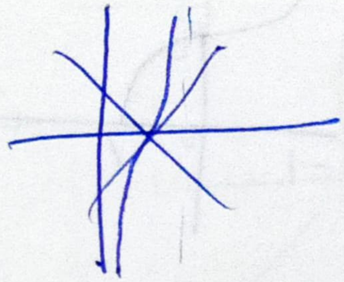
1) sinx



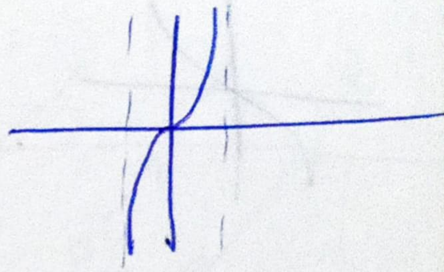
2) cos x



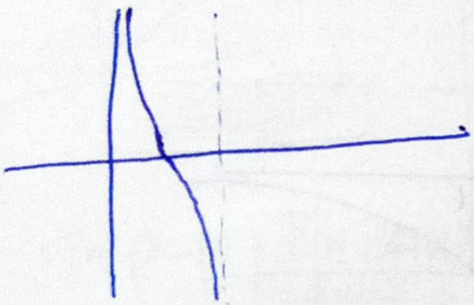
~~3) tanx~~



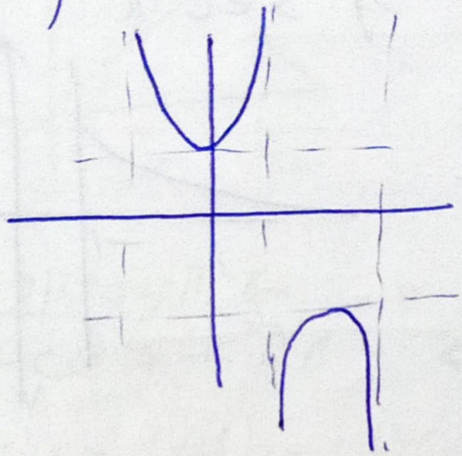
3) tanx



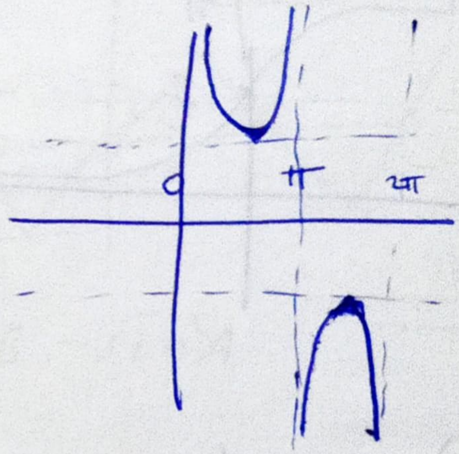
4) cotx



5) Sec X



6) cosecx



<u>Principle values</u>

$\sin\theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $\cos\theta$ $(0, \pi)$, $\tan\theta$ $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\sin\theta = \sin\alpha$ $\qquad$ $\cos\theta = \cos\alpha$ $\qquad$ $\tan\theta = \tan\alpha$

$\boxed{\theta = n\pi + (-1)^n\alpha}$ $\qquad$ $\boxed{\theta = 2n\pi \pm \alpha}$ $\qquad$ $\boxed{\theta = n\pi + \alpha}$

<u>Solutions</u> = distinct root $\quad$ (Ex: $(x-1)(x-1)(x-2) = 0$

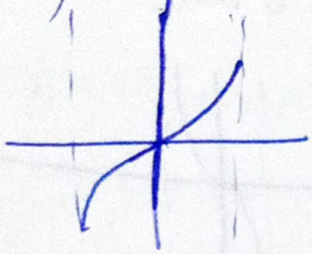
$n(r) = 3 \quad n(s) = 2$

<u>Extraneous solutions</u>: Extra solution which were added while solving & which need to be removed

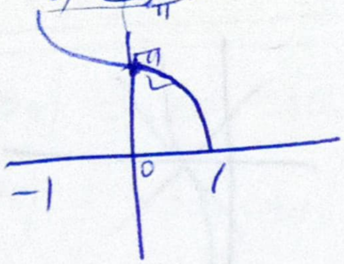
<u>Ex</u>: $\sqrt{1+\sin 2\theta} = \sqrt{2}\cos\theta$

It we directly square we have to remove θ s.t $\cos\theta < 0$ at the end.

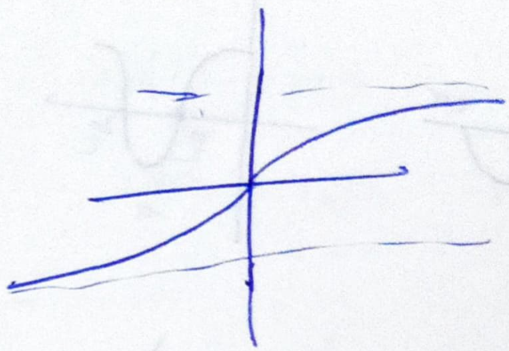
1) $\sin^{-1}x$



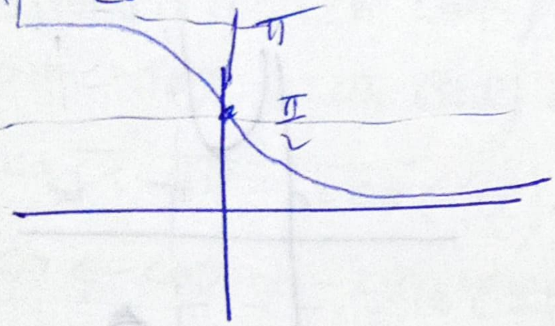
2) $\cos^{-1}x$



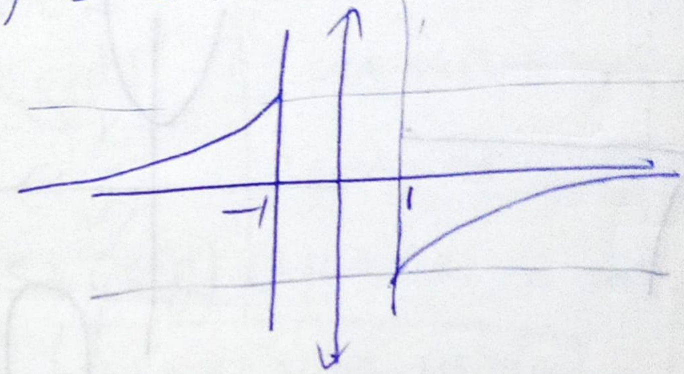
3) $\tan^{-1}x$



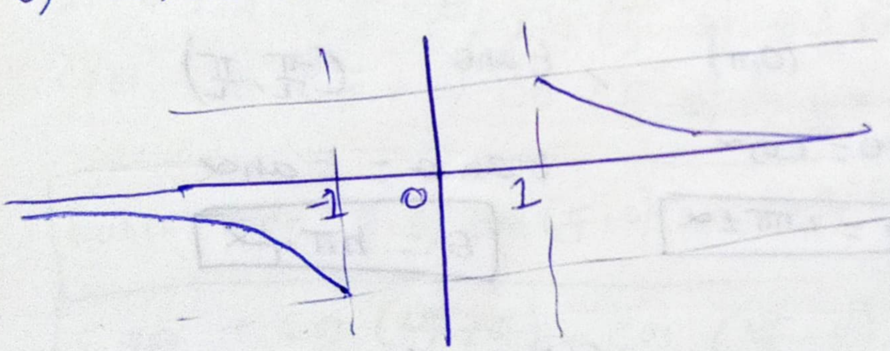
4) $\cot^{-1}x$



5) $\sec^{-1}x$



6) $\operatorname{cosec}^{-1}x$



$\tan^{-1}(-x) = -\tan^{-1}x$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\sec^{-1}(-x) = \pi - \sec^{-1}x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$

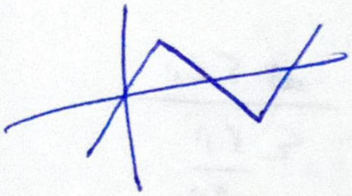
$\sin^{-1}(-x) = -\sin^{-1}x$

$\rightarrow \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ $x, y > 0$ & $xy < 1$

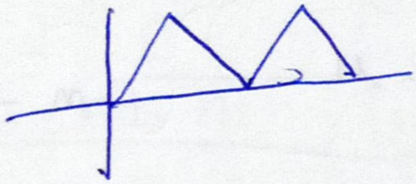
$\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ $x > 0, y > 0$ & $xy > 1$

$-\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ $x < 0, y < 0$ $xy > 1$

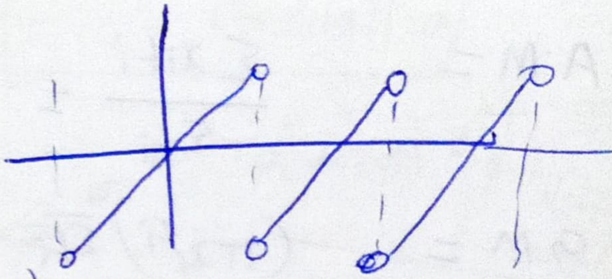
1) $\sin^{-1}(\sin x)$



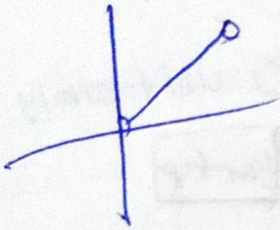
2) $\cos^{-1}(\cos x)$



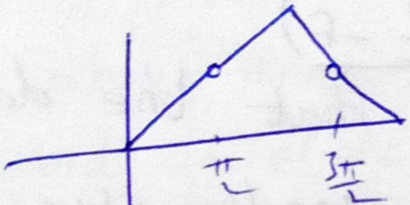
3) $\tan^{-1}(\tan x)$



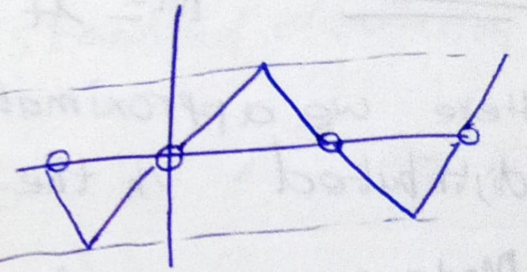
4) $\cot^{-1}(\cot x)$



5) $\sec^{-1}(\sec x)$



6) $\operatorname{cosec}^{-1}(\operatorname{cosec} x)$



$\Rightarrow \sin(A-B)\,\sin(A+B) = \sin^2 A - \sin^2 B$

$\Rightarrow \cos(A-B)\,\cos(A+B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

$\Rightarrow \sin(2\alpha)\,\sin(2\beta) = \sin^2(\alpha+\beta) - \sin^2(\alpha-\beta)$

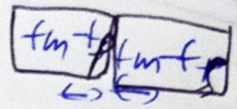
Statistics

A.M = $\frac{\sum x_i f_i}{\sum f_i} = \bar{x} = \mu$ H.M = $\frac{\sum f_i}{\sum \frac{f_i}{x_i}}$

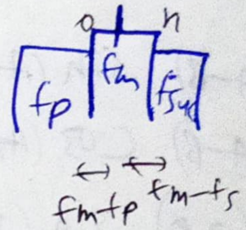
G.M = $\left(\prod x_i^{f_i}\right)^{\frac{1}{\sum f_i}}$

Median: $M = l + \frac{h\left(\frac{N}{2} - F\right)}{f}$

Here we approximate that the data is uniformly distributed in the the median class



Mode: $l + \left(\frac{f_m - f_{preceding}}{2f_m - f_{pre} - f_{seceeding}}\right) h$



Here we approximate that the mode is nearer to the class whose frequency is more

$\Rightarrow$ ☆ $\boxed{\begin{aligned} Z\ (\text{mode}) &= 3M - 2\mu \\ &= 3M - 2\bar{x} \end{aligned}}$

Range: difference between max & min

coeff of range = $\frac{\text{Range}}{\text{Max} + \text{Min}}$

Mean deviation:

about mean = $\frac{\sum |x_i - \mu|}{n}$ (don't forget modulus)

median = $\frac{\sum |x_i - M|}{n}$

mode = $\frac{\sum |x_i - Z|}{n}$

Variance (σ^2)

$\boxed{\sigma = \text{standard deviation}}$

$$V = \sigma^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

$$\boxed{\sigma^2 + \mu^2 = \frac{\Sigma x_i^2}{n}}$$

$$\sigma^2 = \frac{n_1(\sigma_1^2 + (\bar{x}_1 - \bar{x})^2) + n_2(\sigma_2^2 + (\bar{x}_2 - \bar{x})^2)}{n_1 + n_2}$$

$\bar{x}$ = combined mean

Mean deviation about mean = $\frac{4}{5}$ standard deviation

Co-effi of standard deviation = $\frac{\sigma}{\bar{x}}$

Co-eff of variation $= \frac{\sigma}{\bar{x}} \times 100$

Mathematical Reasoning

Statement : It can be either true or false but not both.

Disjunction (Union) $(\vee)$: $p \vee q$

Conjunction (Intersection) $(\wedge)$: $p \wedge q$

Let $p \Rightarrow q$ be a ~~statement~~ Implication or Conditional statement ($p \Rightarrow q = F$ only when $\boxed{p = T, q = F}$
then its

converse is $q \Rightarrow p$

inverse is $\sim p \Rightarrow \sim q$

Contrapositive $\sim q \Rightarrow \sim p$

$\boxed{\text{Biconditional is } p \Leftrightarrow q}$

$\Rightarrow$ $p \vee \sim p$ is tautology

$\Rightarrow$ $p \wedge \sim p$ is contradiction

$\Rightarrow$

1) $(p \vee q) \vee r = p \vee (q \vee r)$
2) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

1) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
2) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

$\Rightarrow$

1) $\sim (p \vee q) = \sim p \wedge \sim q$
2) $\sim (p \wedge q) = \sim p \vee \sim q$

De Morgan's law

$\Rightarrow$ ☆ $\sim (p \Rightarrow q) = p \wedge \sim q$

$\sim (p \Leftrightarrow q) = p \Leftrightarrow \sim q = \sim p \Leftrightarrow q$

Permutations & Combinations

$^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$ & $\lfloor 0 = 1$

$^nP_r = \frac{\lfloor n}{\lfloor n-r}$

Multiplication principle (and)

$m \times n \times \cdots$

Addition principle (or)

$m + n +$

→ exponent of p in $\lfloor n$ (only for primes)

$E_p(n) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \cdots$

→ divisibility by 11 = $abcd = a(11-1)^3 + b(11-1)^2 + c(11-1) + d$

$= -a + b - c + d + 11k$

→ by 7 $\quad abcde = (abcd) \times 10 - 20e + 21e$

$= (abcd - 2e)10 + 21e$

subtract 2 times the last digit from the remaining (If it is divisible)

→ No. of permutations of n objects taken all at a time where p objects like of 1st kind, 'q' objects alike of 2nd kin, r objects alike of 3rd kind is

$\frac{\lfloor n}{\lfloor p \lfloor q \lfloor r}$

$r\,{}^{n-1}P_{r-1} + {}^{n-1}P_r = {}^nP_r$

${}^{n-1}P_r$: excluding

$r\,{}^{n-1}P_{r-1}$: including particular thing

$r(r-1)\,{}^{n-2}P_{r-2} + 2r\,{}^{n-2}P_{r-1} + {}^{n-2}P_r = {}^nP_r$

⇒ No. of permutation of n identical things taken 'r' at a time & each thing can be repeated any no. of times

⇒ 〃 combinations $= {}^nC_1\left({}^{r-1}C_0\right) + {}^nC_2\left({}^{r-1}C_1\right) + \cdots {}^nC_n\left({}^{r-1}C_{r-1}\right)$

$= {}^nC_{n-1}\left({}^{r-1}C_0\right) + {}^nC_{n-2}\left({}^{r-1}C_1\right) + \cdots + {}^nC_0\left({}^{r-1}C_{r-1}\right)$

$= \boxed{{}^{n+r-1}C_{n-1}}$

→ $A_1 A_2 A_3 A_4 A_5$, B_1, B_2, B_3, B_4, B_5 are ~~strand~~ arranged such that As are not side by side

O B_1 O B_2 O B_3 O B_4 O B_5 O $\boxed{{}^6C_5 \times \lfloor 5 \ \lfloor 5}$

→ circular arrangment of n persons $\lfloor n-1$

→ circular arrangment of n beads $\frac{\lfloor n-1}{2}$

→ No. of lines formed by joining n points where no 3 are collinear $= ({}^nC_2)$

→ No. of lines formed by n pts where m pts are collinear $= {}^nC_2 - {}^mC_2 + 1$

→ No. of $\triangle$les by n pt where m pts are collinear ${}^nC_3 - {}^mC_3$

→ No. of regions into which a set of n coplanar lines (no 3 concurrent) divides a plane $= 1 + \frac{n(n+1)}{2}$

→ Consider a convex n-gon

a) → No. of diagnols $= {}^nC_2 - n$
$= \frac{n(n-3)}{2}$

→ No. of $\triangle$le formed by joining 3 vertices is ${}^nC_3$

→ No. of $\triangle$les sharing 1 common side with polygon $= n(n-4)$

2 common $= n$

no common $= {}^nC_3 - n(n-4) - n$

→ n-sided regular polygon

$n = 2m$ →
- → no. of right angled $\triangle$le $= 2m(m-1) = 2\,{}^mP_2$
- → no. of obtuse angle $\triangle$le $= \frac{2 \times {}^{m-1}C_2 \, 2m}{2}$ $= 2m\,{}^{m-1}C_2$
- → No. of acute angled $= {}^{2m}C_3 - 6\,{}^mC_3 - 2m\,{}^{m-1}C_2$

$n = 2m+1$ →
- → No. of right angled $\triangle$le $= 0$
- → No. of obtuse angled $\triangle$le $= \frac{2\,{}^mC_2\,(2m+1)}{2}$
- → No. of acute angled $\triangle$le $= {}^{2m+1}C_3 - {}^mC_2\,(2m+1)$

→ No. of points of intersection of 2 diaghols which lie inside a convex poly-gon (assuming no diagnol are concurrent)
$= {}^nC_4$ (every quadrilateral gives 1 pair of diaghols intersect)

→ n co-planar lines – no 2 parallel – no 3 concurrent
• No. of fresh lines $= {}^{^nC_2}C_2 - n\,({}^{n-1}C_2)$

→ No. of ways of selecting r consecutive objects from 'n' objects $= n-r+1$

→ In a rectangular grid of $p \times q$
→ total no. of squares $= \sum_{r=1}^{\min(p,q)} (p-r+1)(q-r+1)$

→ p distinct objects, q alike of 1st kind, r alike of 2nd kind, s alike of 3rd kind.
No. of combinations $= 2^p (q+1)(r+1)(s+1)$

→ $N = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_n^{\alpha_n}$ No. of Divisors $= (1+\alpha_1)(1+\alpha_2)\dots(1+\alpha_n)$

→ No of proper divisors $= (1+\alpha_1)(1+\alpha_2)\dots(1+\alpha_n) - 2$

→ sum of all divisors $= (p_1^0 + p_1^1 + p_1^2 + \dots + p_1^{\alpha_1})(p_2^0 + p_2^1 + \dots + p_2^{\alpha_2}) \dots (p_n^0 + p_n^1 + p_n^2 + \dots + p_n^{\alpha_n})$

→ Catalan Number $= \frac{{}^{2n}C_n}{n+1}$ = No. of ways of arranging $n \boxed{\uparrow}$ & $n \boxed{\rightarrow}$ S.T at every point $\boxed{\rightarrow}$ should be more than $\boxed{\uparrow}$

→ Derangements No. of ways of arrangin n objects S.T no one is going to the correct place.

$$D_n = (n-1)(D_{n-1} + D_{n-2}) \Rightarrow D_n - nD_{n-1} = -(D_{n-1} - (n-1)D_{n-2}) = (-1)^n$$

$$\boxed{D_n = \lfloor n \left(1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \dots + \frac{(-1)^n}{\lfloor n}\right)}$$

→ Principle of inclusion & exclusion

$$n\left(\bigcup_{i=1}^{n} A_i\right) = \sum n(A_i) - \sum n(A_i \cap A_j) + \sum n(A_i \cap A_j \cap A_k)$$

(D_n can be proved from P-I-E)

Euler's Phi function ($\phi(n)$) : The no. of +ve integers less than or equal to n being relatively prime with

$$\phi(n) = n - \left(\frac{n}{p_1} + \frac{n}{p_2} + \cdots \frac{n}{p_m}\right) + \left(\frac{n}{p_1 p_2} + \frac{n}{p_2 p_3} + \cdots\right) \quad n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_m^{\alpha_m}$$

$$= n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\left(1 - \frac{1}{p_3}\right) \cdots \left(1 - \frac{1}{p_n}\right)$$

No. of onto functions $\quad f : a \to b$

$$n\left(\bigcap_{i=1}^{n} A_i^c\right) = n(S) - n(A_1 \cup A_2 \cup \cdots A_r)$$

$$= r^n - \left({}^{r}C_1 (r-1)^n - (r-2)^n\, {}^{r}C_2 + (r-3)^n\, {}^{r}C_3 - \cdots\right)$$

$$= b^a - {}^{b}C_1 (b-1)^a + {}^{b}C_2 (b-2)^a - {}^{b}C_3 (b-3)^a - \cdots$$

short cut $\Rightarrow$

{1, 2, 3, 4, 5} → {b_1, b_2, b_3}

$3^5 - {}^3C_1 2^5 + {}^3C_2 (1)^5$ or

b_1	b_2	b_3	
2	2	1	${}^5C_2 \times {}^3C_2 \times \frac{3!}{2!}$
1	1	3	${}^5C_3\, {}^2C_1\, \frac{3!}{2!}$

$\rightarrow \quad x_1 + x_2 + \cdots + x_n = r, \quad x_i \in N \quad r \geq n$

$\Rightarrow \boxed{{}^{r-1}C_{n-1}}$

$\rightarrow \quad x_1 + x_2 + \cdots + x_n = r \quad x_i \in W$

$${}^{n+r-1}C_{n-1} = \boxed{{}^{r+n-1}C_{n-1}}$$

$\rightarrow \quad x_1 + x_2 + \cdots + x_n \leq r$

then transform it into $\quad x_1 + x_2 + \cdots + x_n + k = r$

Grouping

$$\frac{\lfloor x_1 + x_2 + x_3 + \cdots x_n}{\lfloor x_1 \lfloor x_2 \cdots \lfloor x_n}$$

mn objects $\rightarrow$ $\boxed{\dfrac{\lfloor mn}{(\lfloor n)^m \lfloor m}}$

Catalan Number: ${}^{2n}C_n - {}^{2n}C_{n+1} = \frac{{}^{2n}C_n}{n+1}$ (only going on one side of triangle)

Total paths without touching diagnol $= 2\times\left(\frac{{}^{2n}C_n}{n+1}\right)$

Proof 1

$a_1 + a_2 + \dots a_k \geq 0 \quad \forall k \in [1, 2n], k \in I$

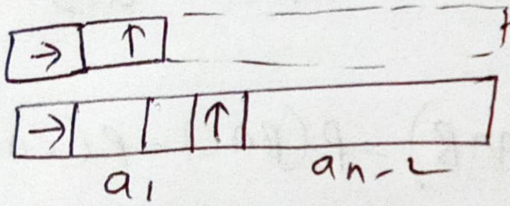
let it is violated in a case. In all those violated case let k be the first term at which $a_1 + \dots a_k \leq 0 \Rightarrow k = \text{odd}$

$(a_1, \dots a_{k-1}) \in \left(\frac{k-1}{2}(+) \text{ \& } \frac{k-1}{2}(-ve)\right)$

fore every such violated series we can replace a_i with $-a_i \ \forall i \in (0, k]$ then the sum of this series is $\boxed{+2}$ implies it is number ways of arrangin $(n+1)$ +ve & $(n-1)$ -ve

$= {}^{2n}C_{n+1}$

Proof 2:



$\boxed{a_0 = 1}$

$a_n = a_0 a_{n-1} + a_1 a_{n-2} + \dots a_{n-1} a_0$

$\Rightarrow$ ~~an~~ let $f(x) = a_0 + a_1 x + \dots a_n x^n$

coef of x^n in $f(x) = x^{n-1}$ in $f(x)^2 \quad \forall n \geq 1$

$f(x) - a_0 = x f(x)^2$

$\Rightarrow f(x) = \frac{1 \pm \sqrt{1-4x}}{2x} = \frac{2}{1+\sqrt{1-4x}}$

$$\boxed{a_n = \frac{{}^{2n}C_n}{n+1}}$$

Probability

Random experiment : repeatable experiments - results aldready known - exact results not known.

Mutually exclusive events : if occurense of 1 prevents the ~~other~~ occurence of all other (or exclusive)

Pair wise mutually ex clusive $A_i \cap A_j = 0$

Exeaustive events

$$\boxed{\bigcup_{r=1}^{n} E_r = S}$$

odds in favour of $A = a:b$

$\Rightarrow P(A) = \frac{a}{a+b}$

odds ~~in favour of~~ against $A = a:b$

$P(A) = \frac{b}{a+b}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)$$

$\Rightarrow$ **Pack set** 52 cards

Diamond & heart (Red) spade & Club (black

Each contains $2, 3, 4, 5, 6, 7, 8, 9, 10, \underbrace{\underbrace{J, K, Q}_{\text{face cards}}, A}_{\text{honour cards}}$, Aces cards

$$P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)} = \frac{P(A \cap B)}{P(A)}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

Independent event $\Rightarrow P(A \cap B) = P(A)\,P(B)$

→ mutally independent ⇒ $P(A \cap B) = P(A)P(B)$ & $P(A \cap B \cap C) = P(A)P(B)P(C)$

→ Pairwise " ⇒ $P(A \cap B) = P(A)P(B)$

Total Probability Theorem

$$P(A) = \sum_{i=1}^{n} P(E_i) \cdot P\left(\frac{A}{E_i}\right)$$

(they are mutually exclusive & exhaustive events)

Bayes theorem

$$P\left(\frac{E_K}{A}\right) = \frac{P(E_K)\,P\left(\frac{A}{E_K}\right)}{\sum_{i=1}^{n} P(E_K)\,P\left(\frac{A}{E_K}\right)}$$

Binomial Probability

$$P(x=r) = {}^{n}C_{r}\,(q)^{n-r}\,p^{r}$$

Properties of triangles

$a+b+c=2R$

Sine Rule

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$$

Projection Rule

$$a=b\cos C+c\cos B$$
$$b=c\cos A+a\cos C$$
$$c=a\cos B+b\cos C$$

Cosine rule

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

or

$$a^2=b^2+c^2-2bc\cos A$$

Molliweide's rule

$$\frac{a+b}{c}=\frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}\qquad \frac{a-b}{2}=\frac{\sin\left(\frac{A-B}{2}\right)}{\cos\frac{C}{2}}$$

Tangent rule or Napier's Analogy

$$\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\frac{C}{2}$$

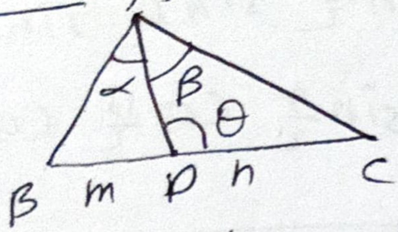
Half angle formulas

$$\sin\left(\frac{A}{2}\right)=\sqrt{\frac{(s-b)(s-c)}{bc}}\qquad \cos\left(\frac{A}{2}\right)=\sqrt{\frac{s(s-a)}{bc}}$$

$$\tan\left(\frac{A}{2}\right)=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\frac{\Delta}{s(s-a)},\quad \cot\left(\frac{A}{2}\right)=\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}=\frac{\Delta}{(s-b)(s-c)}$$

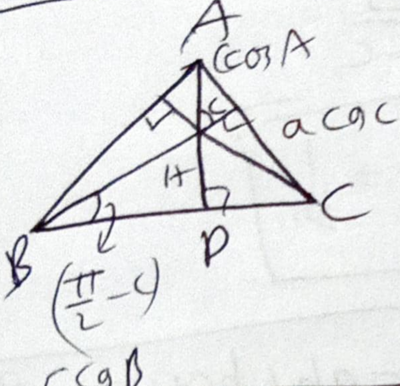
$$\boxed{\Delta=\sqrt{s(s-a)(s-b)(s-c)}\quad \text{Heron's formula}}$$

m, n cot theorem



$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$$
$$= n\cot B - m\cot C$$

Distances from Orthocentre



$AH = 2R\cos A$

$HD = 2R\cos B\cos C$

$$\boxed{\frac{AH}{\sin A} = \frac{c\cos A}{\sin C}}$$

$$\boxed{\frac{HD}{\cos C} = \frac{c\cos B}{\sin C}}$$

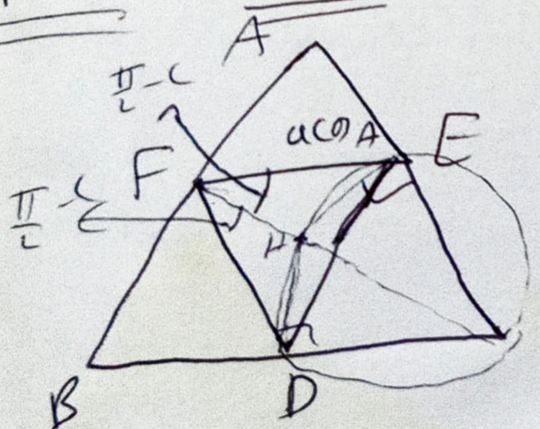
$$\boxed{\cot A = \frac{b^2+c^2-a^2}{4\Delta}}$$

$$\boxed{O \;{}^{3}\; N \;{}^{1}\; G \;{}^{2}\; S}$$

$SA = SB = SC = R$ $\quad SD = R\cos A$ $\quad SE = R\cos B$ $\quad SF = R\cos C$

$\boxed{D, E, F = \perp\text{lars from } S}$

Pedal Triangle



angle in the same segment is equal.

$\angle DHC = \angle DEC =$

$\angle DHC = B \Rightarrow \boxed{\angle DEC = B}$

sides are $a\cos A, b\cos B, c\cos C$

$R_{pedal} = \frac{R}{2}$

$s = 4R\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ $\qquad$ $r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

$r_1 - r = 4R\sin^2\frac{A}{2}$

$r_2 + r_3 = 4R\cos^2\frac{A}{2} \Rightarrow \boxed{r_1 + r_2 + r_3 - r = 4R}$

$r = \frac{\Delta}{s}$ $\quad r_1 = \frac{\Delta}{s-a}$ $\quad r_2 = \frac{\Delta}{s-b}$ $\quad r_3 = \frac{\Delta}{s-c}$

$\boxed{\Delta = \sqrt{r r_1 r_2 r_3}}$ $\qquad$ $\boxed{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}}$

$\boxed{r_1 r_2 + r_2 r_3 + r_3 r_1 = s^2}$ $\qquad$ $\boxed{r(r_1 + r_2 + r_3) = ab + bc + ca - s^2}$

$$\boxed{\begin{aligned} SI &= \sqrt{R^2 - 2Rr} \\ SI_1 &= \sqrt{R^2 + 2Rr_1} \end{aligned}}$$

$$\boxed{\begin{aligned} HS &= R\sqrt{1 - 8\cos A\cos B\cos C} \\ SI &= \sqrt{R^2 - 2Rr} \\ HI &= \sqrt{2r^2 - 4R^2\cos A\cos B\cos C} \end{aligned}}$$

$\cancel{r^2 = r_1 r_2}$

→ If $f(0)=0$ & $f(x)>0$ $\forall x \in (0,1)$

$f(x)\,f(1-2x)=0$; $x=0, \frac{1}{2}$

$\Rightarrow \frac{f'(c)}{f(c)} = \frac{2f'(1-2c)}{f(1-2c)}$

$\boxed{\text{there exist a } c}$

also

$f(x)\sqrt{f(1-2x)}$

$\frac{f'(c)}{f(c)} = \frac{f'(1-2c)}{f(1-2c)}$

$\in \boxed{\text{there exist a } c}$

→ $g(x) = \begin{cases} \left(5^{\frac{x}{3}} - 5^{-\frac{x}{3}}\right)^5 \cos(\sin x^2) & x \in R - I \\ \left[\frac{1}{2}\sin^2 x + \frac{3}{10}\right] & x \in I \end{cases}$

g is an odd function $\left([\;] = 0\right]$

→ Look at domain carefully while calculating number of discontinous points

→ $\sqrt{1+3x} + \sqrt{3+5x} + \sqrt{5+x} = \sqrt{3-x} + \sqrt{5-3x} + \sqrt{1-5x}$

increasing function — decreasing function

either 1 root or no roots

So

→ $I = \int_0^{\frac{\pi}{2}} \tan^3 x \cos^5 x \cos 7x \, dx$

$I = \int_0^{\frac{\pi}{2}} \sin^3 x \cos^3 x \cos 7x \, dx$

$I = -\int_0^{\frac{\pi}{2}} \sin^3 x \cos^3 x \sin 7x \, dx$

$\boxed{\text{add them}}$

→ $\int_{-5}^{-2} \left(\frac{x^2-x}{x^3-3x+1}\right)^2 dx + \int_{\frac{1}{6}}^{\frac{1}{3}} \left(\frac{x^2-x}{x^3-3x+1}\right)^2 dx + \int_{\frac{6}{5}}^{\frac{3}{2}} \left(\frac{x^2-x}{x^3-3x+1}\right)^2 dx =$

$\int_{-5}^{-2} \left(x+1-\frac{1}{x}-\frac{1}{x-1}\right)^{-2} \left(1+\frac{1}{x^2}+\frac{1}{(x+1)^2}\right) dx = \int_{-5}^{-2} \left(\frac{x^2-x}{x^3-3x+1}\right)^2 \left(1+\frac{1}{x^2}+\left(\frac{1}{x-1}\right)^2\right) dx$

$\rightarrow \quad \dfrac{\underset{x\to\infty}{Lt}\left(1+\dfrac{x^2}{\lfloor 2}+\dfrac{x^4}{\lfloor 4}+\dots\right)}{\underset{x\to\infty}{Lt}\left(x+\dfrac{x^3}{\lfloor 3}+\dots\right)} = l$

applying L-Hopital rule

$$\frac{x+\frac{x^3}{\lfloor 3}+\dots}{1+\frac{x^2}{\lfloor 2}+\dots} = \frac{1}{l} \Rightarrow \boxed{l=\frac{1}{l}} \quad \boxed{l=1}$$

(or)

$$\underset{x\to\infty}{Lt}\, e^{-x} = \underset{x\to\infty}{Lt}\left(1-x+\frac{x^2}{\lfloor 2}-\frac{x^3}{\lfloor 3}\right)=0$$

$\rightarrow \quad S=\displaystyle\sum_{k=0}^{\infty}\frac{2^k}{7^{2^k}+1} = \sum_{k=0}^{\infty}\frac{2^k}{7^{2^k}+1}\times\frac{7^{2^k}+1-2}{7^{2^k}-1}$

$$=\boxed{\sum_{k=0}^{\infty}\frac{2^k}{7^{2^k}-1} - \frac{2^{k+1}}{7^{2^{k+1}}-1}}$$

$\rightarrow \quad x = \dfrac{\sin 65^\circ}{\sqrt{3-2\cos 70-2\cos 15\sqrt{2-2\cos 70}}} = \dfrac{\sin 65}{\sqrt{3-2\cos 70-2(\sin 50-\sin 80)}}$

$$=\frac{\sin 65}{\sqrt{3-2\cos 70-1+2\sin 80}}$$

$$=\frac{\sin 65}{\sqrt{2+2\cos 10-2\cos 70}}$$

$$=\frac{\sin 65}{2\cos(25)}=\frac{\sin 65}{\sqrt{2+2\cos 50}}=\frac{\sin 65}{\sqrt{2+2\sin 40}}$$

→

1	2	1.414
	1	
24	100	
	96	
	400	
281	281	
	11900	
2824	11296	
	3004	

→

1	3	1.732
	1	
27	200	
	189	
343	1100	
	1029	
3462	7100	
	6924	
	176	

→

2	5	2.236
	4	
	100	
42	84	
	1600	
443	1329	
	27100	
4466	26796	
	304	

→ $\cos x\left(\sin x+\sqrt{\sin^2 x+\tfrac{1}{2}}\right) = \cos x\sin x + \cos x\sqrt{\sin^2 x+\tfrac{1}{2}}$

$\leq \sqrt{\cancel{\textcircled{a}} \times \tfrac{3}{2}}$

$\left(\cos x\sin x+\cos x\sqrt{\sin^2 x+\tfrac{1}{2}}\right)^2 \leq \tfrac{3}{2}\times(1)$

(Cauchy-swartz inequality)

$\dfrac{\cos x}{\sin x} = \dfrac{\sqrt{\sin^2 x+\tfrac{1}{2}}}{\cos x}$

→ $a+f''(x)+f'(x)=x^2+f^2(x)$. If p is the point of maximum of $f(x)$ then no. of tangents drawn from p to $x^2+y^2=a$ is

$f'(x)=0 \quad f''(x)<0$ at $p \Rightarrow \boxed{x^2+f^2(x)-a<0}$

$\rightarrow \left|z+\frac{1}{z}\right| = 2 \Rightarrow z = r\cos\theta + ir\sin\theta$

$\Rightarrow \left(r+\frac{1}{r}\right)^2\cos^2\theta + \left(r-\frac{1}{r}\right)^2\sin^2\theta = 4$

$4\cos^2\theta + \left(r-\frac{1}{r}\right)^2 = 4$

$(x^2+y^2-1)^2 = 4y^2$ $\Leftarrow 4(r\cos\theta)^2 + (r^2-1)^2 = 4r^2$

$\rightarrow$ similarily $\left|z+\frac{k}{z}\right| = \sqrt{4k}$ al $\Rightarrow$ 2 circles

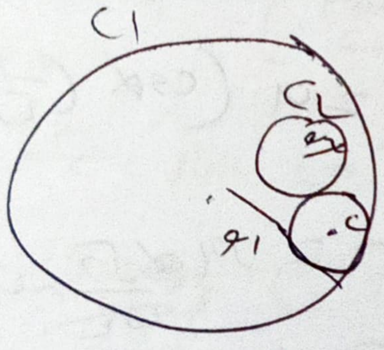
$\rightarrow \int_0^2\left(\sqrt{1+x^3} + \sqrt[3]{x^2+2x}\right)dx = \int_0^{2}\left(1+\sqrt{1+x^3}\right) + \sqrt[3]{x^2+2x}\,dx$

$= 2+\int_0^2$ $\quad$ ~~$f(x) = \sqrt[3]{x^2+2x}$~~ ~~$f^{-1}(x) =$~~

$= 2 + f(a)\,a - b\,f(b)$ (substitute $f(a)$ & $f(b)$ carefully)

$\Rightarrow$ whenever given points – A, B, C, P – read carefully which is A, which is B

$\rightarrow$



$CC_2 = r+r_2$

$CC_1 = r_1 - r$

$|CC_1 + CC_2| = r_1 + r_2$

$\rightarrow \frac{dy}{dx} = \frac{y^3}{e^x + y^2} \Rightarrow d\left(e^{-x}y\right) = d\left(\frac{1}{y}\right)$

$\rightarrow$ a unit vector making equal angle with x, y, z = $\cancel{\frac{\hat{i}+\hat{j}+\hat{k}}{3}}$

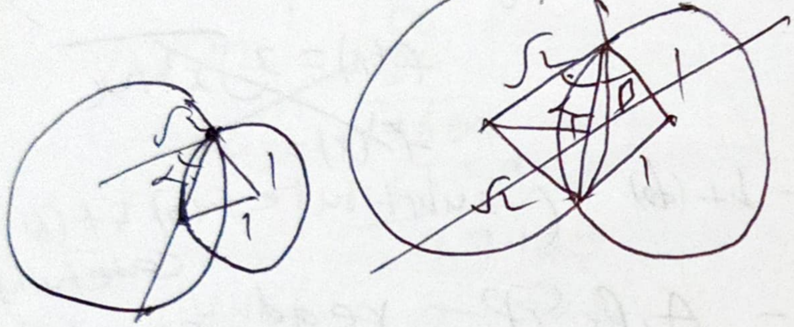
$= \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

$\rightarrow \sum_{r=1}^{n}\sqrt{1+\frac{1}{r^2}+\frac{1}{(r+1)^2}} = \sum_{r=1}^{n}\sqrt{1+\frac{1}{(r+1)^2}-\frac{2}{r+1}+\frac{1}{r^2}} = \sum_{r=1}^{n}\sqrt{\left(1-\frac{1}{r+1}\right)^2+\frac{1}{r}\frac{2}{r+1}}$

$$= \sqrt{\left(1-\frac{1}{r+1}\right)^2 + \frac{1}{r^2} - \frac{2}{r+1}}$$

$$= \Sigma \; 1 - \frac{1}{r+1} + \frac{1}{r}$$

→ A circle with radius $\sqrt{2}$ & another with 1 meet at A such that through A a chord ~~at~~ can be drawn which is bisected by the circle with radius 1. Then the length of the chord is



$$\cos(\alpha+\beta) = \frac{-1}{2\sqrt{2}} = \cos\theta$$

$$\frac{x}{2} = \sqrt{2}\cos\alpha$$

$$\frac{x}{4} = \cos\beta$$

$$2\sqrt{2}\cos\alpha = 4\cos(\theta-\cancel{\alpha}$$

$$2\sqrt{2}\cos\alpha = 4\left(\cos\alpha\left(\frac{-1}{2\sqrt{2}}\right) + \sin\alpha \times \frac{\sqrt{7}}{2\sqrt{2}}\right)$$

$$3\cancel{\sqrt{2}}\cos\alpha = \sin\alpha \frac{\sqrt{7}}{\cancel{\sqrt{2}}} \times 4$$

~~$\cos\alpha$~~

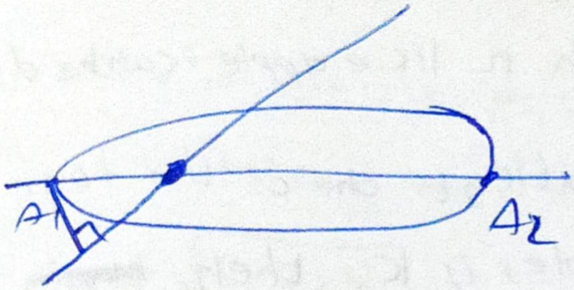
$$\boxed{\tan\alpha = \frac{3}{\sqrt{7}}}$$

$$\frac{x}{2} = \sqrt{2} \times \frac{\sqrt{7}}{4}$$

$$\boxed{x = \sqrt{\frac{7}{2}}}$$

⇒ If $a^2+b^2+c^2+2abc = 1$ & one of them is a proper fraction

~~a^2+bc~~ $(a+bc)^2 = 1-b^2-c^2+b^2c^2 = (1-b^2)(1-c^2) \geq 0 \Rightarrow \boxed{\begin{matrix} b^2<1 \\ c^2<1 \\ a^2<1 \end{matrix}}$

→  To find the minimum distance of a point on the

→ If the volume of tetrahedron ABCD is a

$\angle ACB = \frac{\pi}{6}$, $2AD + AC + BC = 18$, then AD=

$a \leq \frac{1}{3}(AD)\left(\frac{1}{2} \times AC\,BC \times \frac{1}{2}\right) \Rightarrow AC\,BC\,AD = 3^3 \times 2$

$(2AD)(BC)(CB) = 3^3 \times 2^2$

$\boxed{2AD = BC = CB = 6}$

→ $\int_0^{\pi/2} \theta \cot\theta = \left[\theta \log(\sin\theta)\right]_0^{\pi/2} - \int \log(\sin\theta)\, d\theta$

But $\boxed{\int_0^{\pi/2} \log(\sin\theta)\, d\theta = -\frac{\pi}{2}\log 2}$

→ $\lim_{n\to\infty} n^2\left(\frac{\tan^{-1}(n+1)}{n+1} - \frac{\tan^{-1}(n)}{n}\right) = \lim_{x\to 0} \frac{1}{x^2}\left(\frac{\tan^{-1}(1+\frac{1}{x})}{1+\frac{1}{x}} - \frac{\tan^{-1}(\frac{1}{x})}{\frac{1}{x}}\right)$

→ $\cos^5 x + \sin x \cos^4 x - \sin^4 x + \sin^3 x + \cos x \sin^2 x - \cos^2 x = 0$

$\cancel{(\sin x + \cos x - 1)(\cos^4 x + \sin^2 x) = 0}$ $\quad (\underbrace{\sin x + \cos x - 1}_{\boxed{0}})(\cos^4 x + \sin^2 x) = 0$

→ A is a diagnol matrix $\quad$ B is a diagnol matrix

$A^7 + 3A^5 + 7A = 11I$ $\quad B^5 = I$, (atleast 1 non zero real element)

$x = 1$ $\quad y = 5^3 - 4^3$

→ From $(k, 0)$ to $x^2 + y^2 = 16$ – 3 chords – those chords are bisected by $y^2 = 2x$, $k > 4$ then no. of values of $k = \boxed{1}$

$x\left(\frac{t^2}{2}\right) + y(t) = k\frac{t^4}{4} + t^2$ $\quad$ $A\left(\frac{t^2}{2}, t\right)$ lies inside the circle

$k\left(\frac{t^2}{2}\right) = \frac{t^4}{4} + t^2$

→ The no. of ways in which n like apple can be distributed to 4 persons A, B, C, D so that atleast one of the four persons gets exactly 3 apples is k then ~~...~~

$$k = {}^4C_1\left({}^{11}C_2\right) - {}^4C_2\left({}^7C_1\right) + {}^4C_3(1) - {}^4C_4(1)$$

$= 4 \times 11 \times 5 - 6(7) + 3$
$\;\;\; 220 \qquad\;\; 42$
$\qquad\qquad \to 9 \quad = \boxed{181}$

→ what is the largest 2 digit prime factor of ${}^{200}C_{100} = 61$

~~$101 \times 102 \times 103 \times$~~ . $\cancel{\dfrac{1 \times 3 \times 5 \times \cdots 199}{1 \times 2 \times 3 \times \cdots \times 100}} \times 2^{100} = \cancel{\dfrac{51 \times 53 \times \cdots 199}{1 \times 3 \times \cdots \times 50}} \times 2^{50}$

$\dfrac{101 \times 102 \times 105 \cdots 199}{1 \times 3 \times \cdots \times 49} \times 2^{50}$ $\quad \dfrac{200}{3} = 66 \quad \boxed{61} \to$

→ **Domain:**
Domain
Domain
Domain
Domain →

$f(x) > ,\; x \in (0, \pi)$

$f(x) - 1 = x^2$ no. of solutions of $x = \cancel{x}\ \boxed{1}$

$f \geq \left[0\ \tfrac{\pi}{2}\right] \to [0, 1] \quad f(0) = 0 \quad f\left(\tfrac{\pi}{2}\right) = 1$

$\boxed{(f(x))^2}$ $\quad 2f(\alpha)\, f'(\alpha) = \dfrac{1}{\frac{\pi}{2}} \Rightarrow \boxed{f(\alpha) f'(\alpha) = \frac{1}{\pi}}$

$f(x) - \dfrac{x^2}{\left(\frac{\pi}{2}\right)^2}$ $\quad g' \quad f'(\cancel{\alpha}) = f'(\alpha) = \dfrac{8\alpha}{\pi^2}$

→ $g(x) = \displaystyle\int_x^{\frac{\pi}{2}} \left(f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t\, f(t)\right) dt$

$\boxed{\lim_{x \to 0} g(x) \neq g(0)}$

→ $x^2+y^2+2x+4y-p=0$ find no. of p s.t it touches the co-ordinate axes at exactly 3 points = $\cancel{3}$ 2

→ $$e^{-\sqrt{|\ln\{x\}|}} - \{x\}^{\sqrt{\frac{1}{|\ln\{x\}|}}} = \frac{\ln\{x\}}{e^{\sqrt{\ln\{x\}}}} - \{x\}^{\frac{1}{\sqrt{\ln\{x\}}}}$$

$$= \boxed{0}$$

→ $$s(n) = \sum_{r=1}^{n} \frac{\cos(2\cdot 3^{r-1}\alpha)}{\sin(3^r\alpha)}, \quad \alpha = 18^\circ, \quad 5\alpha = \frac{\pi}{2} \Rightarrow 2\alpha = \frac{\pi}{2} - 3\alpha$$

$$\Rightarrow s(n) = \sum_{r=1}^{n} \frac{\cos(3^{r-1}(\frac{\pi}{2}-3\alpha))}{\sin(3^r\alpha)} = 1-1+1-1 \ldots\ldots$$

$$= \cancel{\frac{(-1)^n - n}{2}} = \frac{1^n - (-1)^n}{2}$$

$$P(n) = \sum_{r=1}^{n} \frac{\cos(3\cdot 2^{r-1}\beta)}{\cos(2^r\beta)}, \quad \beta = 36, \quad 5\beta = \pi$$

$$3\beta = \pi - 2\beta$$

$$= \sum_{r=1}^{n} \frac{\cos(2^{r-1}(\pi - 2\beta))}{\cos(2^r\beta)} = \sum_{r=1}^{n} \frac{\cos(2^{r-1}\pi - 2^r\beta)}{\cos(2^r\beta)}$$

$$= -1+1 \ldots$$

$$= n-2$$

→ $$P = \sum_{n=1}^{\infty} {}^{2n}C_n \frac{1}{5^n} = \sum_{r=1}^{n} \frac{1\times 3\times 5 \ldots (2n-1)}{\lfloor n} \times \frac{2^n}{5^n}$$

$$= \sum_{r=1}^{n} \frac{\frac{1}{2}\times\frac{3}{2}\times\frac{5}{2}\ldots\left(\frac{2n-1}{2}\right)}{\lfloor n}\left(\frac{4}{5}\right)^n$$

$$= \left(1-\frac{4}{5}\right)^{-\frac{1}{2}} = \boxed{\sqrt{5}}$$

→ ${}^{50}C_n$, $(1 \le n \le 49)$ is not always divisible by 50.

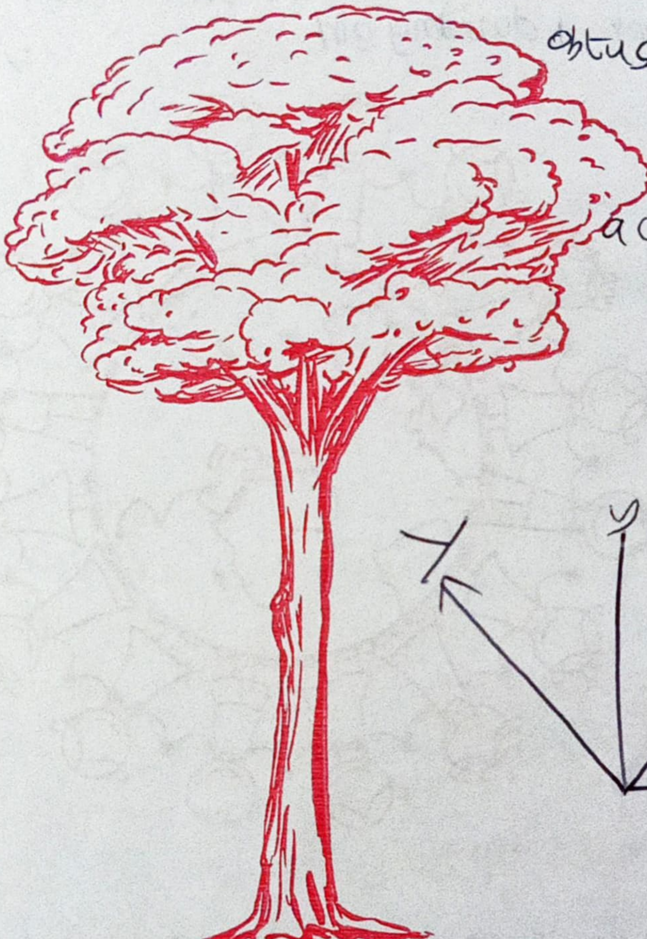
Did you know?

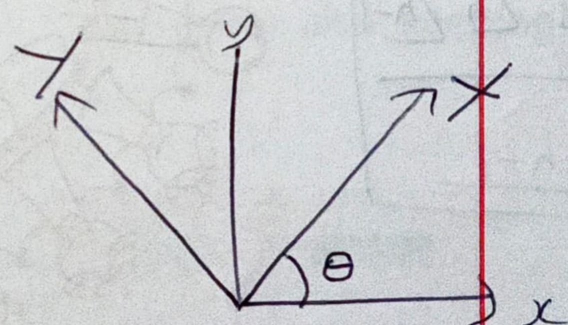
$c_1 > 0, c_2 > 0$

$a_1a_2 + b_1b_2 > 0$

obtuse $\frac{L_1}{\sqrt{a_1^2+b_1^2}} = \frac{L_2}{\sqrt{a_2^2+b_2^2}}$

acute $\Rightarrow \frac{L_1}{\sqrt{a_1^2+b_1^2}} = -\frac{L_2}{\sqrt{a_2^2+b_2^2}}$





	X	Y
$x-h$	$\cos\theta$	$-\sin\theta$
$y-k$	$\sin\theta$	$\cos\theta$

The tallest tree on Earth is called Hyperion and measures 379.1 feet.

The Earth's moon is called Luna. It is the brightest object in the sky after the sun.

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$$\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5}}}}\cdots = \boxed{3}$$

$$\sqrt{1+x\sqrt{1+(1+x)\sqrt{1+(x+1)\sqrt{\;}}}} = 1+x$$

Earth Facts!

n boys (n-1) girls no. boys ahead of every girl is atleast 1 more than n (boys)

(consider 1 dummy girl

$$\Rightarrow \frac{\frac{{}^{2n}C_n}{n+1}\lfloor n \lfloor n-1}{\lfloor 2n-1}$$

Wrong method

$\square \rightarrow$ 1 boy is fixed

(n-1) boys (n-1) girls

$$\boxed{\frac{\frac{{}^{2(n-1)}C_{n-1}}{n}\lfloor n \lfloor n-1}{\lfloor 2n-1}}$$

Reason

in the above we took only those cases infront of a girls atleast 2 more than

$$\sum \frac{4r}{4r^4+1} = \sum \frac{4r}{(2r^2+1)^2-4r^2}$$

$$= \sum \frac{4r}{(2r^2-2r+1)(2r^2+2r+1)}$$

How old is the Earth? Earth is about five billion years old! There are more than seven billion people on Earth and the number keeps increasing every day.

The distance between the Earth and the sun is 150 million kilometres.

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How do we begin recycling garbage?

We can start recycling garbage by sorting it out.

How do we do that? We can put the used materials in separate bins for plastic, paper, glass, organic (examples: food which cannot be eaten, fish bones, weeds in your garden, egg shells, dirty paper, and dried leaves), and general (items that cannot be put in the other bins) things.

$X \quad Y$

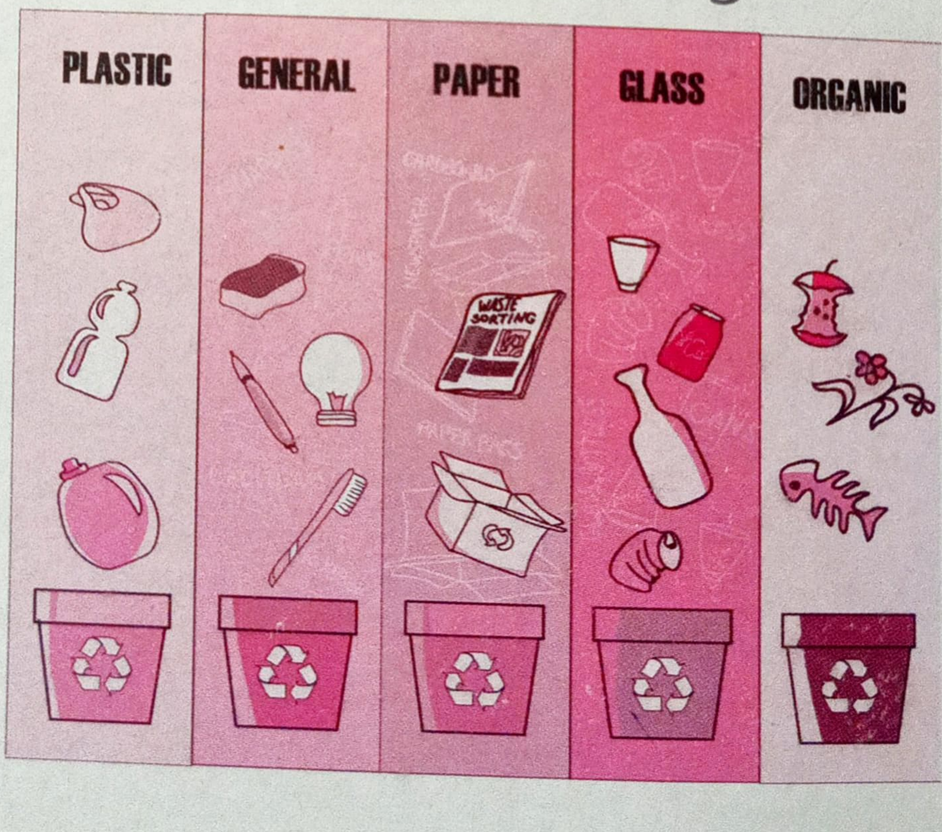
$x-h \quad \cos\theta \quad -\sin\theta$

$y-k \quad \sin\theta \quad \cos\theta$

(use comple numbers)

By throwing garbage into the bins, we are also keeping our surroundings and environment clean.

$\int$



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