Higgs mechanism

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Gauge theory

Recall that in classical EM $A^\mu = \left(\frac{\phi}{c}, \mathbf{A}\right)$ has the following redundancy

$$A_\mu \to A_\mu - \partial_\mu \chi.$$

So, why did we introduce this fake symmetry instead of just using **E** and **B**?

1. We are trading a less symmetric description for a more symmetric but redundant one
2. Manifest Lorentz invariance
3. Better suited to quantize
4. When fake symmetry is not useful, we can gauge fix and remove it
5. More elegant (next slide)

Gauge group

In a more elegant way, "gauging" a global symmetry will imply the existence of a gauge field. For example, for an electron, the action is

$$\mathcal{S} = \int \bar{\psi}\,(i\,\gamma^{\mu}\partial_{\mu} - m)\,\psi\,\mathrm{d}^4 x$$

The global symmetry for this system is

$$\psi \mapsto e^{i\theta}\psi$$

"Gauging" or "Localising": $\theta \to \theta(x)$ demanding this symmetry implies the existence of the electromagnetic field A_{μ}

$$S_{\text{QED}} = \int d^4x \, \left[\bar{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m)\,\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - eJ^{\mu}A_{\mu}\right]$$

We can specify a theory by its gauge group, for example, the $SO(10)$ theory.

Yang–Mills theory (1954)

Generalization of EM: What if the gauge symmetry is $SU(n)$ (n-dimensional unitary matrices) instead of $U(1)$ (unit complex numbers)?

$SU(n)$ is non-abelian (non-commutative).

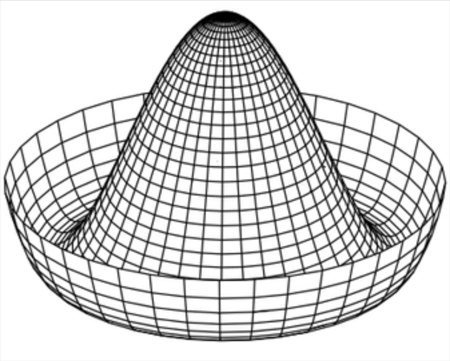
Standard model is a $SU(3) \times SU(2) \times U(1)$ theory.

When Yang and Mills discovered the Yang-Mills theory, Pauli criticized their theory because it predicted only massless particles. Pauli himself worked earlier on some non-abelian gauge theory and stopped because of this. Naively adding the mass term will violate gauge invariance $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m^2 A_\mu^2$. Massless particles should be easy to observe. So, the possible answers that Yang-Mills theory is physically realizable are

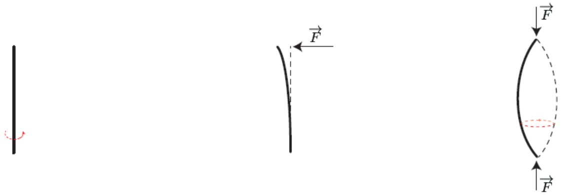
1. The Yang-Mills particles acquire mass through some mechanism and is only observable at high energies
2. The Yang-Mills particles are, in fact, massless but are not observed because the theory doesn't allow them to be detected.

The 1st possibility is realized in the electroweak theory ($SU(2)$) through the **Higgs mechanism**, and the 2nd possibility is realized as **color confinement** in QCD ($SU(3)$). We will discuss the Higgs mechanism. It is also called **ABEGHHK'tH** mechanism (for Anderson(1962), Brout, Englert, Guralnik, Hagen, Higgs, Kibble, and 't Hooft).

Spontaneous symmetry breaking is a spontaneous process of symmetry breaking by which a physical system in a symmetric state spontaneously ends up in an asymmetric state. More precisely, spontaneous symmetry breaking happens when the ground state doesn't have the full symmetry of the theory.



Graph of Goldstone's "sombrero" potential function $V(\phi)$.



(a) Unbroken symmetry: the rod in its original state is rotationally invariant

(b) Explicitly broken symmetry: the rod bends due to an external force and loses rotational invariance

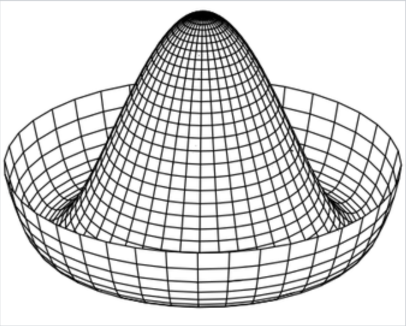
(c) Spontaneously broken symmetry: the rod bends in an arbitrary direction and loses rotational invariance

Figure 1: Spontaneous symmetry breaking as seen in the bending of a rod

Assume the radius of the rod is 0 for simplicity.

Nambu–Goldstone bosons (1960)

Goldstone's theorem: whenever a continuous **global** symmetry is spontaneously broken, massless fields, known as Nambu-Goldstone bosons, emerge. **Nobel Prize**.



Graph of Goldstone's "sombrero" potential function $V(\phi)$.

The motion along the circle with minimal potential will give a new degree of freedom that behaves like a massless boson.

Abelian Higgs mechanism (1964)

The Lagrangian for a complex scalar coupled to a $U(1)$ gauge field is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^* D^\mu\phi - V(|\phi|),$$

where $D_\mu = \partial_\mu + ieA_\mu$ The gauge transformation is

$$\phi(x) \to e^{-i\alpha(x)}\phi(x),$$

$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x).$$

We choose the most general analytic, gauge-invariant, renormalizable potential (implies powers more than 4 are not contained since they have couplings with negative mass dimension) with a symmetry-breaking term

$$V = -m^2 \phi^* \phi + \frac{\lambda}{4} (\phi^* \phi)^2$$

or we can add a constant and define (which will not contribute as we neglect gravity)

$$V = \frac{\lambda}{4} \left(|\phi|^2 - \frac{v^2}{2} \right)^2$$

The wrong-sign mass term for the scalar indicates that the ground state has $|\langle\phi\rangle| = \frac{v}{\sqrt{2}} = \sqrt{\frac{2m^2}{\lambda}}$. All the vacua are equivalent (by symmetry) so we can pick any convenient parametrization. Let us take $\langle\phi\rangle$ to be real. Now expand around v by parametrizing $\phi(x)$ in terms of two real fields $\sigma(x)$ and $\pi(x)$ as

$$\phi(x) = \left(\frac{v + \sigma(x)}{\sqrt{2}}\right) e^{i\frac{\pi(x)}{v}}$$

Plugging this in our Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \left(\frac{v+\sigma}{\sqrt{2}}\right)^2 \left[-i\frac{\partial_\mu \pi}{v} + \frac{\partial_\mu \sigma}{v+\sigma} - ieA_\mu\right]\left[i\frac{\partial_\mu \pi}{v} + \frac{\partial_\mu \sigma}{v+\sigma} + ieA_\mu\right]$$
$$- \left(-\frac{m^4}{\lambda} + m^2\sigma^2 + \frac{1}{2}\sqrt{\lambda}m\sigma^3 + \frac{1}{16}\lambda\sigma^4\right)$$

Now look at the terms involving only A_μ

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2v^2A_\mu^2 + \cdots$$

we see $m_A = ev$ from the square term. So, now the **gauge field has acquired mass**. Similarly

$$m_\sigma = \sqrt{2}m$$

$$m_\pi = 0$$

This is informally stated as "gauge fields become massive by the Higgs mechanism, after they **eat** a massless scalar degree of freedom (Nambu–Goldstone boson) along the symmetry direction in order to become massive".
The guage transformation

$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x), \quad \pi(x) \to \pi(x) - v\alpha(x)$$

will not change the Lagrangian. So, the gauge symmetry is not broken.

Correct interpretation

As is usual, people who first come up with an equation or concept usually do not give the correct interpretation. Example: Lorentz transformations (correct interpretation by Einstein), Schrodinger equation (correct interpretation by Born), Dirac equation (he interpreted it as wave-function instead of a quantum field)
This situation is called "spontaneous breaking of a gauge symmetry," which is a **misnomer**. The local gauge symmetry remains a gauge symmetry; the choice of vacuum doesn't affect the fact that states related by a gauge transformation are physically the same. **Actually breaking gauge symmetry would be disastrous**; it occurs in the case of a gauge anomaly and destroys the Ward identities, **making the theory inconsistent**. **Breaking the global symmetry does not violate gauge symmetry** since we require gauge transformations to vanish at infinity.

Elitzur's theorem (1975): implies that gauge symmetry cannot be spontaneously broken. Only the corresponding global part can be spontaneously broken.

Intuitively, SSB is a physical phenomenon, but the gauge redundancy is not physical and was introduced by us to make things easier. Obviously, it won't make sense for some physical phenomena to remove the redundancy.

Gauge fixing

Unitary gauge or **physical gauge**: $\alpha(x) = \frac{\pi(x)}{v} \implies \pi(x) = 0$. In this gauge the particle $\pi(x)$ doesn't exist. Since physics should be independent of the particular gauge we chose, we can say that π is **not a real particle**. It is called as physical gauge because it removes the unphysical **ghost Goldstone π particle**.

R_ξ-gauges: one adds a gauge breaking term to the "physical" (gauge invariant) Lagrangian

$$\delta\mathcal{L} = -\frac{(\partial_\mu A^\mu)^2}{2\xi}$$

The choice of the parameter ξ determines the choice of gauge. In this gauge the theory satisfies both conditions needed for renormalizability:

1. The propagator of any bosonic field falls off like $1/p^2$ for large momenta and the propagator of any fermion field falls off like $1/p$.
2. There is no coupling constant with negative mass dimension.

2 more Nobels

That is why the gauge condition with an arbitrary finite value of ξ is called the renormalizable ξ-gauge or the R_ξ-gauge.
For finite ξ, the theory is renormalizable, although the particle interpretations are somewhat obscure because of the presence of unphysical bosons.
On the other hand, for infinite ξ (physical gauge), the particle spectrum is clear, although renormalizability is not obvious.
Non-Abelian Higgs mechanism: In real life electroweak symmetry $SU(2) \times U(1)_Y$ breaks to $U(1)_{em}$ as discovered by Weinberg (1967), Glashow & Salam. Its very similar to the Abelian model we saw. **Nobel Prize**.
1972: 't Hooft and Veltman showed that electroweak theory (and other gauge theories) is renormalizable. **Nobel Prize**.

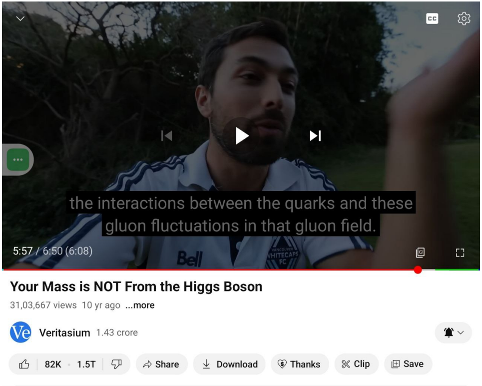
Noether's theorem

Gauge symmetries do not give any (non-trivial) conservation laws, unlike their global counterparts. You might **incorrectly** guess that conservation laws will become invalid as they are derived from global symmetries.

Fabri-Picasso theorem: When a global symmetry is spontaneously broken, the corresponding conserved charge does not exist because its correlation functions are IR divergent. However, the conserved current and even the commutators with the conserved charge do exist.

Intuitively, You can look around this room and see that rotational symmetry is broken, but angular momentum is still conserved in everyday life.

Popular science misinformation



Every elementary particle (except possibly neutrinos) gets its mass from the Higgs field. But still, most of your mass (or proton's mass) is actually from **fluctuations in the gluon field**. Also, **the God particle** didn't prove that gods exist and are made up of them. Leon Lederman understandably hyped it to get funding after SSC was canceled (check the Anderson vs Weinberg debate if interested).

References

Check my old notes https://ksr.onl/files/HiggsNotes.pdf, or the below books

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