Holographic description of M-theory

Sreeman Reddy Kasi Reddy

Brandeis University
Final presentation for 287A course at Harvard

7 Dec 2023

Table of Contents

Recall the 5 string theories

	IIB	IIA	$E_8 \times E_8$	$SO(32)$	I
String Type	Closed	Closed	Closed	Closed	Closed + open unoriented
No. of SUSY generators	2 (chiral)	2 (non-chiral)	1 (chiral)	1 (chiral)	1 (chiral)
Gauge Groups	$U(1)$	$U(1)$	$E_8 \times E_8$	$SO(32)$	$\mathrm{SO}(32)$
D-branes	$-1, 1, 3, 5, 7, 9$	$0, 2, 4, 6, 8$	none	none	$1, 5, 9$

$p+1$-form gauge fields (field strength $p+2$ form) naturally couple to extended objects with $p+1$ dimensional world-volume (p-branes in SUGRA and D-p-branes in ST). $p = 0 \implies$ familiar EM case.

$11D$ SUGRA is unique: only (local) SUSY theory in $11D$ containing only particles of spin ≤ 2. All other SUGRA can be obtained by dimensional reduction. Resurrected due to M theory.
Type IIA string theory at strong coupling becomes an 11-dimensional theory called M-theory, whose low energy is the 11-D SUGRA, with the fields $g_{\mu\nu}$, a rank 3 anti-symmetric tensor field $C_{\mu\nu\rho}$ and a 32 component Majorana gravitino Ψ_μ^α. Gauge invariance under $C \to C + d\Lambda$ where Lambda is a 2-form. The Lagrangian is given by

$$
\begin{aligned}
I_{11} = & \frac{1}{16\pi G_N^{(11)}} \int d^{11}x \sqrt{-g^{(11)}} \left[R_{(11)} - \frac{1}{2.4!} G^2 - \frac{1}{2} \bar{\Psi}_\mu \Gamma^{\mu\nu\rho} D_\nu(\Omega) \Psi_\rho \right. \\
& \left. - \frac{1}{192} \left(\bar{\Psi}_\mu \Gamma^{\mu\nu\rho\lambda\sigma\tau} \Psi_\tau + 12 \bar{\Psi}^\nu \Gamma^{\rho\lambda} \Psi^\sigma \right) G_{\nu\rho\lambda\sigma} \right] \\
& - \frac{1}{96\pi G_N^{(11)}} \int C \wedge G \wedge G + \text{ terms quartic in } \Psi
\end{aligned}
$$

where $G = dC$ is the field strength of C and Ω_μ^{ab} is the spin connection, which appears in the covariant derivative $D_\nu(\Omega)\Psi_\rho = \left(\partial_\nu - \frac{1}{4}\Omega_\nu^{ab}\Gamma_{ab}\right)\Psi_\rho$.

M-branes

A simple way to see this is the KK dimensional reduction. Which gives

$$R = g_s\sqrt{\alpha'},$$

As expected, the 11th dimension disappears as $g_s \to 0$. Note that there is no coupling constant in M-theory.
Two p-brane solutions

1. $G = G_4 \implies$ couples to a 2-brane. Low energy limit of M2-brane.
2. $\star G_4 = G_7 \implies$ couples to a 5-brane. Low energy limit of M5-brane

Under the electric-magnetic duality, $M2$ and $M5$ (magnetically charged) are dual to each other. Recall Hodge star operator

$$*\left(\mathrm{d}x^{i_1}\wedge\mathrm{d}x^{i_2}\wedge\cdots\wedge\mathrm{d}x^{i_p}\right)=\frac{1}{(n-p)!}e^{i_1\ldots i_p}{}_{i_{p+1}\ldots i_n}\,\mathrm{d}x^{i_{p+1}}\wedge\cdots\wedge\mathrm{d}x^{i_n}$$

$M2$ brane is the fundamental object in M theory. We know how to quantize particle (QM), string (ST), but it is harder to quantize a $2+1$ object. $M2$ can become either string or $D2$ brane under KK.

M-theory	IIA
wrapped membrane	IIA string
unwrapped membrane	IIA D 2-brane
wrapped 5-brane	IIA D 4-brane
unwrapped 5-brane	IIA NS 5-brane
KK photon $(g_{\mu 11})$	RR gauge field A_μ
quantized momentum $p_{11} = n/R_{11}$	IIA 0-branes

M theory has mostly the same properties as string theory: UV complete. Unitary, **Holographic**, Unique. Except in this case the uniqueness is not hidden under non-trivial S and T dualities like in $10D$ string theories. Since string theories are special cases of M theory, there might be new features in M theory

Open/closed string perspectives

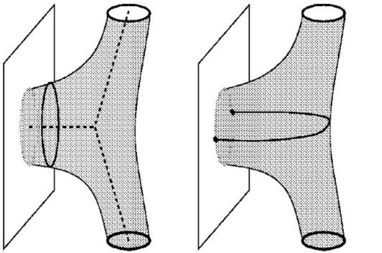

Fig. 2. There are two ways to think about the interaction between closed strings and brane.

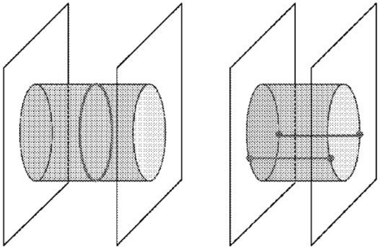

Fig. 4. There are two ways to think about the interaction between two branes. One involves opens strings, the other is mediated by closed string modes.

N coincident $D3$-branes in type IIB

Open string perspective:
Recall from the last class that a single $D3$ brane with DBI action (it is similar to Maxwell field with UV cutoff) has $U(1)$ symmetry, and for coincident $D3$ branes, the gauge group is enhanced to a $U(N)$ gauge theory.
- The 6 transverse coordinates become six real scalar fields ϕ_i (6 DoF) in the world volume description. The $U(N)$ gauge field has 2 DoF. We need 4 Majorana fermions to have world volume SUSY. Then we can see that what we have is $\mathcal{N} = 4SYM$ on the world volume.
Open string perspective: $\mathcal{N} = 4$ Super Yang-Mills theory on flat four-dimensional spacetime and type IIB supergravity on $\mathbb{R}^{9,1}$. ($g_s N << 1$)
This is called the decoupling of the world volume theory from the bulk.

Closed string perspective

The type IIB supergravity background created by a stack of N coincident $D3$-branes can be written as:

$$ds^2 = h(r)^{-1/2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + h(r)^{1/2}\left(dr^2 + r^2 d\Omega_5^2\right),$$

$$h(r) = 1 + \frac{L^4}{r^4}, \quad L^4 = 4\pi g_s N \alpha^2,$$

$$g_s F_5 = (1 + *) d^4x \wedge dh^{-1}(r).$$

As can be seen from the metric, in the "**near horizon limit**," which corresponds to taking $r \ll L$, the term $h(r)$ can be approximated by L^4/r^4 and, therefore, it is easy to see that the geometry of space-time becomes that of $AdS_5 \times S^5$. Thus, the system of N D3-branes in the near horizon limit is described by a Type IIB string theory in $AdS_5 \times S^5$ space-time.
Closed string perspective: type IIB supergravity on $AdS_5 \times S^5$ and type IIB supergravity on $\mathbb{R}^{9,1}$. ($g_s N >> 1$)

- In both perspectives, in decoupling (low-energy) limit, the same system is a sum of two noninteracting systems. In both descriptions, one of the two systems was free IIB supergravity. It is, therefore, natural to expect that the two remaining systems are equivalent.
- S and T dualities were surprising and non-trivial. This is even more surprising because unlike S-duality, which can connect 2 QFTs or 2 String theories, this duality connects a String Theory and a Gauge Theory. 2 very **different frameworks**. Since nature likes gauge theories and used them as the basis of the standard model, it gives theoretical (not empirical) evidence that string theory is correct.
- 't Hooft and Susskind guessed this based on the black hole entropy formula.
- **Conformal boundary of AdS interpretation**: In addition, the boundary values of fields in the bulk theory source operators in the field theory in the dictionary of the correspondence. Therefore, it makes sense to think of the gauge theory as being located at the boundary. It is not there by construction!

Statement

$\mathcal{N} = 4$ $SU(N)$ SYM with Yang-Mills coupling constant g_{YM} is dynamically equivalent to type IIB superstring theory with string length $l_{\mathrm{s}} = \sqrt{\alpha'}$ and coupling constant g_{s} on $AdS_5 \times S^5$ with radius of curvature L and N units of $F_{(5)}$ flux on S^5.

The two free parameters on the field theory side, i.e. g_{YM} and N, are mapped to the free parameters g_{s} and $L/\sqrt{\alpha'}$ on the string theory side by

$$g_{\mathrm{YM}}^2 = 2\pi g_{\mathrm{s}} \quad \text{and} \quad 2g_{\mathrm{YM}}^2 N = L^4/\alpha'^2.$$

Fix $\lambda = g_{\mathrm{YM}}^2 N$ and apply large N limit to get classical theory. Large λ give SUGRA limit.

AdS dictionary	CFT
Z_{AdS}	Z_{CFT}
m of bulk scalar fields	Δ of scalar primary operators
Asymptotically AdS geometry	Quantum state
Ex: Pure AdS	Vacuum State
Ex: 2-sided Schwarzschild AdS BH	Thermofield Double State
Single string excitations	Single trace operators
Gauge fields	Conserved currents
$g_{\mu\nu}$	$T_{\mu\nu}$
IR divergence	UV divergences
Radial direction	Energy scale
Branes	Solitons
D(-1)-instanton	instanton VEV
Wrapped branes	Baryonic vertex
RT or QES	von Neumann entropy
Entanglement wedge	ρ of a subregion
Action	Complexity
Cosmic Brane	Rényi entropies

The metric for N extremal M5-branes is

$$ds^2 = f_5^{-1/3}(r)\left(-dt^2 + d\vec{x}_{(5)}^2\right) + f_5^{2/3}(r)\left(dr^2 + r^2 d\Omega_4^2\right),$$

$$f_5(r) = 1 + \frac{\pi N l_P^3}{r^3}.$$

l_P =11-dimensional Planck length. There is also a 4-form field strength,

$$F_{(4)} = *\left(dt \wedge dx^1 \wedge \ldots \wedge dx^5 \wedge d\left(f_5^{-1}\right)\right);$$

$$\left(A_{(6)} = f_5^{-1} dt \wedge dx^1 \wedge \ldots \wedge dx^5\right),$$

where $A_{(6)}$ is the Poincaré dual to the 3-form gauge field $A_{(3)}$.
- Take the near-horizon limit to decouple gravity from field theory modes along the brane, in order to have a duality between string theory in a gravitational background and the field theory existing on the brane. We define

$$U^2 \equiv \frac{r}{l_P^3}$$

and apply $l_P \to 0$ and $r \to 0$, with $U =$ fixed.

The metric becomes

$$ds^2 = l_P^2 \left[\frac{U^2}{(\pi N)^{1/3}} \left(-dt^2 + d\vec{x}_{(5)}^2 \right) + 4(\pi N)^{2/3} \frac{dU^2}{U^2} + (\pi N)^{2/3} d\Omega_4^2 \right],$$

which is the metric of $AdS_7 \times S^4$, With radii $R_{\mathrm{S}^4} = l_p(\pi N)^{1/3}$ and $R_{AdS7} = 2l_p(\pi N)^{1/3} = 2R_{\mathrm{S}^4}$.

Brane world volume theory: The bosonic subgroup of the symmetries of this supergravity solution is $SO(6,2) \times SO(5)$. The dual field theory is a six-dimensional conformal field theory with R-symmetry group $SO(5)$. The theory preserves sixteen Poincaré supercharges which may be grouped into two left-handed supersymmetry generators transforming in the spinorial representation $\mathbf{4}_l$ of SO(5). The theory is therefore known as N = (2, 0) theory. It has no Lagrangian description.

- When we compactify such that $M5$ becomes $D4$, we get 5-dimensional maximally supersymmetric Yang-Mills theory with $g_{Dp}^2 = (2\pi)^{p-2} g_s {\alpha'}^{\frac{p-3}{2}} \Rightarrow g_{D4}^2 = 4\pi^2 R_{11}$. Since it depends only on R_{11} we can say that 6D theory has no dimensionless coupling.

$AdS_4 \times S^7/\mathbb{Z}_k$ vs 3D $\mathcal{N}=6$ $U(N)\times U(N)$ SCS theory

- We need to find the theory living on coincident $M2$-branes. For $11-3=8$ transverse directions, we get 8 scalars. If we do KK reduction to get $D2$-branes, the fields will change to $10-3=7$ scalars and 1 $U(N)$ gauge field. Since this is just changing the coupling value, the no. degrees of freedom should not change. They have the same degrees of freedom.
- But we need to add some gauge field in $M2$-brane to get the $U(N)$ gauge field after KK. Since the degrees of freedom already matched, we can only add something like Chern-Simons term which has no degrees of freedom in $3D$. To have SUSY we further need to add 4 Majorana fermions.
- So, the fields are 4 complex bosons C^I, four Majorana fermions ψ_I and CS gauge field A^μ.
- It turns out that we need a gauge group $SU(N)\times SU(N)$ instead of the usual $SU(N)$, which means that there are CS gauge fields A_μ and $\hat{A}_\mu$ for the two $SU(N)$ factors, and C^I and ψ_I are bifundamental, i.e. transforming as the representation $(N,\bar{N})$ under $SU(N)\times SU(N)$.

Action

$$S_{\mathrm{ABJM}} = \int d^3x [\frac{k}{4\pi}\epsilon^{\mu\nu\lambda}\,\mathrm{Tr}\left(A_\mu\partial_\nu A_\lambda + \frac{2i}{3}A_\mu A_\nu A_\lambda - A_\mu\partial_\nu A_\lambda - \frac{2i}{3}A_\mu A_\nu A_\lambda\right)$$

$$- \mathrm{Tr}\left(D_\mu C_I^\dagger D^\mu C^I\right) - i\,\mathrm{Tr}\left(\psi^{I\dagger}\gamma^\mu D_\mu \psi_I\right)$$

$$+ \frac{4\pi^2}{3k^2}\mathrm{Tr}\left(C^I C_I^\dagger C^J C_J^\dagger C^K C_K^\dagger + C_I^\dagger C^I C_J^\dagger C^J C_K^\dagger C^K\right.$$

$$\left. + 4 C^I C_J^\dagger C^K C_I^\dagger C^J C_K^\dagger - 6 C^I C_J^\dagger C^J C_I^\dagger C^K C_K^\dagger\right)$$

$$+ \frac{2\pi i}{k}\mathrm{Tr}\left(C_I^\dagger C^I \psi^{J\dagger}\psi_J - \psi^{\dagger J} C^I C_I^\dagger \psi_J - 2 C_I^\dagger C^J \psi^{\dagger I}\psi_J + 2\psi^\dagger\right.$$

$$\left. + \epsilon^{IJKL}\psi_I C_J^\dagger \psi_K C_L^\dagger - \epsilon_{IJKL}\psi^{\dagger I} C^J \psi^{\dagger K} C^L\right)] .$$

Here the covariant derivative acts like

$$D_\mu C^I = \partial_\mu C^I + i\left(A_\mu C^I - C^I \hat{A}_\mu\right).$$

$\mathcal{N}=6$ supersymmetry $\Rightarrow$ R-symmetry is $U(1) \times SU(4)_R (SU(4) = SO(6))$. The gauge fields are Chern-Simons type, i.e. $\propto \mathrm{Tr}[A \wedge dA + 2/3 A \wedge A \wedge A]$, whose field equation is $F = dA + A \wedge A = 0$, meaning that there is no propagating degree of freedom. The coefficient of the CS form is $k/4\pi$, with k **quantized** at the quantum level, i.e. $k \in \mathbb{Z}$, for consistency of e^{iS} (so that it is single valued under all gauge transformations). Here the integer k is called the level. Then we see that in the ABJM action the two gauge factors have levels k and $-k$, i.e. $SU(N)_k \times SU(N)_{-k}$.

- Here k serves as the deformation parameter. $1/\sqrt{k}$ is the coupling. Perturbation theory is valid at $k \to \infty$, and the $k=1$ case is strongly coupled, and hard to describe.

M2 to D2 and Mukhi–Papageorgakis Higgs mechanism

In three dimensions, we could have the usual Higgs mechanism, but there is also another version. A Chern–Simons (non-dynamical) gauge field can "eat" a real scalar and become a **dynamical Yang-Mills field** (with one degree of freedom in three dimensions), through the same redefinition as in the usual 4-dimensional case.

$$C^I = \nu \delta^{I4} + z^I .$$

The YM coupling will be

$$\frac{k^2}{32\pi^2\nu^2} \equiv \frac{1}{4g^2_{\Upsilon\mathcal{M}}},$$

$M2$-brane construction

Table 8.1 Type IIB brane construction leading to ABJM theory

	0	1	2	3	4	5	6	7	8	9
NS5	•	•	•	•	•	•	–	–	–	–
NS5′	•	•	•	•	•	•	–	–	–	–
N D3	•	•	•	–	–	–	•	–	–	–
k D5	•	•	•	•	•	–	–	–	–	•

1) Start in type IIB string theory with two $NS5$-branes in the directions 012345, separated in the 6 direction, which is compact. Then consider N D3-branes in the 012 and 6 directions, with the D3-branes wrapping the whole compact direction 6, and thus intersecting both the NS5 and the NS5', as in Fig. a. Thus the common worldvolume is in directions 012, i.e. 3-dimensional, and it is an $\mathcal{N}=4$ susy theory (the system breaks 1/2 susy) with gauge group $U(N)\times U(N)$ (one $U(N)$ factor on each D3-NS5 intersection), with two $\mathcal{N}=2$ chiral multiplets A_i, $i=1,2$ in the $(N,\bar{N})$ representation and two $\mathcal{N}=2$ chiral multiplets $B_j, j=1,2$ in the $(\bar{N},N)$ representation

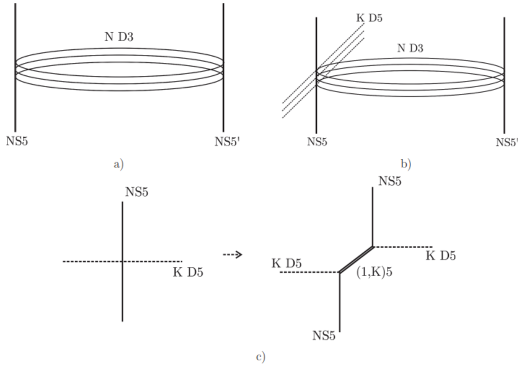

Figure 20.1 a) Brane construction, step 1; b) Brane construction, step 2; c) Brane construction, step 3.

2) Add k D5-branes, intersecting at the same point the NS5-brane and the N D3-branes, in the directions 012349, as in Fig. 20.1b. This breaks the susy to $\mathcal{N} = 2$ and adds k massless chiral multiplets in the fundamental N and k massless chiral multiplets in the antifundamental $\bar{N}$ of each $U(N)$ factor.

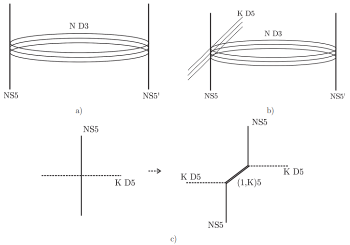

Figure 20.1 a) Brane construction, step 1; b) Brane construction, step 2; c) Brane construction, step 3.

3) Introducing CS gauge fields: One first obtains a mass term for the chiral multiplets by separating the intersection of the NS5-brane and k D5-branes through an intermediate bound state, called a $(1,k)$5-brane, as in Fig. 20.1c. Supersymmetry fixed the angle θ in the $(5,9)$ plane to be given by

$$\theta = \arg(\tau) - \arg(k+\tau); \quad \tau = \frac{i}{g_s} + \chi.$$

Then integrating out the fermions in the chiral and antichiral multiplets gives rise to a CS term (as always in three dimensions), with a level $+1/2$ for each Majorana fermion of positive mass and $-1/2$ for each Majorana fermion of negative mass, for a total level of $+k$ for the first $U(N)$ and $-k$

- The fourth step is to rotate the $(1, k)$5-brane in the (37) and (48) directions by the same angle θ above, obtaining a brane in the $012[37]_\theta[48]_\theta[59]_\theta$ directions, for a total 3-dimensional theory with $\mathcal{N} = 3$ supersymmetry.
- The final step is to T-dualize to type IIA string theory and lift to M-theory. When T-dualizing on direction 6 , we obtain type IIA theory compactified on direction $\tilde{6}$. The D3-brane turns into a D2-brane, since the direction 6 is parallel to the D3-brane. Lifting to M-theory, the D2-brane becomes an M2-brane.

From the brane construction: the IR limit of the system corresponds to the near-horizon limit of N M2-branes probing the $\mathbb{C}^4/\mathbb{Z}_k$ singularity. On the other hand, the IR limit of the field theory, which is the $\mathcal{N} = 3$ supersymmetric $U(N) \times U(N)$ theory with YM-CS gauge fields described before, is exactly the ABJM model, the $\mathcal{N} = 6$ supersymmetric $U(N) \times U(N)$ CS gauge theory.

Gravity dual and $k \to \infty$ limit

The M2-branes probe the $\mathbb{C}^4/\mathbb{Z}_k$ singularity, where $\mathbb{Z}_k$ acts on the eight Euclidean coordinates of $\mathbb{C}^4$ as

$$Z_i \to e^{\frac{2\pi i}{k}} Z_i$$

and since as we saw in the previous chapter, the gravity dual of $N\mathrm{M2}$-branes in flat space is $AdS_4 \times S^7$, the gravity dual in our case is $AdS_4 \times S^7$, divided by the $\mathbb{Z}_k$ action, i.e. $AdS_4 \times S^7/\mathbb{Z}_k$, since the 7-sphere is defined by $\sum_{i=1}^4 |Z_i|^2$.

- But the 7 -sphere can be writtten as an S^1 Hopf fibration over $\mathbb{CP}^3$, and then the $\mathbb{Z}_k$ action just makes the S^1 k times smaller. As $k \to \infty$, the gravity dual becomes $AdS_4 \times \mathbb{CP}^3$ in type IIA string theory.
- The reason S^1 becomes k times smaller is because $e^{i\theta}\mathbb{Z}_k = e^{i(\theta+\frac{2\pi}{k})}\mathbb{Z}_k$

How far is this model from reality?

Pros: compared to the original $AdS_5 \times S^5/\mathcal{N} = 4SYM$

1. M theory is more fundamental. So, a non-perturbative understanding of it is more important than string theories, which are limiting cases.
2. It has the same non-compact spacetime dimensions as reality.

Cons: these are also present in the original example

1. $R_{S^7} = 2R_{AdS_4}$. In general, the compact dimensions are in the same order as R_{AdS_4}, but we need the compact dimensions to be much smaller for a realistic holographic model. (Check 0908.0756 "Small Extra Dimensions in AdS/CFT" by Polchinski and Silverstein)
2. de Sitter space (more precisely *FLRW* with $\Lambda > 0$): although *AdS/CFT* is sufficient to understand quantum gravity in the interior of black holes, we need a holographic description of the universe to understand Big Bang singularity.