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Fundamental Physics

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Abstract: These are some short notes on fundamental physics. These are largely about QM, GR, and QFT. These notes only talk about frameworks already experimentally verified. Things like inflation and beyond the Standard Model topics, etc, which are not yet experimentally verified but use these established frameworks are also included in this. For quantum gravity, check notes at ksr.onl/QG.

Contents

1 Introduction

> The supreme task of the physicist is to arrive at those universal elementary laws from which the **cosmos can be built up by pure deduction**. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them. In this methodological uncertainty, one might suppose that there were any number of possible systems of theoretical physics all equally well justified; and this opinion is no doubt correct, theoretically. But the development of physics has shown that at any given moment, out of all conceivable constructions, a single one has always proved itself decidedly superior to all the rest.
>
> *Albert Einstein* (1918)

Fundamental physics is that part of physics that cannot be reduced to some other physics. If you keep asking "*Why?*" for physical phenomena, you always end up at fundamental physics, and your curiosity must end there as you can't find an answer to "*Why?*" anymore. Fundamental physics is the Big Bad of physics. By definition, the fundamental laws cannot be explained from some other physical explanation. The only type of explanation I can think of is an ontological argument, you can argue that the fundamental laws of physics (possibly a theory of everything) is the greatest possible Platonic mathematical entity, and due to this property it not existing physically is *logically impossible* and therefore it must exist *a priori* without any physical reason for its existence. Any reasonable scientist *must believe in the reductionist philosophy* that every physical phenomenon can be reduced to fundamental physics (including those we haven't understood, such as consciousness). In philosophical words, all physical phenomena supervene on the fundamental laws of physics (possibly a theory of everything). In [1], Anderson **correctly** points out that not everyone who agrees with reductionism must agree that fundamental physics is the most important research direction and explains that you can do highly creative research on emergent phenomena without working in fundamental physics. Anderson also mentions that many great fundamental physicists have used condescending language ("*the discoverer of the positron said "the rest is chemistry"*" here Anderson is talking about the *predictor* P. A. M. Dirac, not the *discoverer* Carl Anderson) to describe applied physics. I want to clarify that even though I am only interested in fundamental physics, I respect all fields and all researchers. My preference for fundamental physics is like my preference for our Indian cuisine compared to other cuisines or my preference for animanga (collective term for anime and manga) over other forms of entertainment, just a personal preference, and I don't claim things I like to be objectively more interesting.

What is considered fundamental physics depends on the time. For example, in Newton's time, his theories *Classical Mechanics* (CM) and *Newtonian Gravity* (NG) were considered fundamental. But today, we know that those 2 are some limits of quantum field theory ($QFT \to QM \to CM$) and general relativity ($GR \to NG$). In the future, we might find that QFT and GR are limiting cases of a theory of everything (the most promising candidate being string theory), then that theory will replace these two as the fundamental theory. Lagrange, who rewrote Newton's theories into his new Lagrangian formulation, famously said,

> Newton was the greatest genius that ever existed, and the most fortunate, for we cannot find more than once a system of the world to establish.
>
> *Joseph-Louis Lagrange*

But Lagrange wrongly thought that fundamental physics was over with CM and NG. **Fortunately for us**, it's not over yet, and the last piece of fundamental physics is likely quantum gravity.

In practice, we must study the limiting cases before studying QFT and GR. [9–19] are books that are similar to these notes. [9–18] follows the same approach as me, starting with old fundamental theories and ending with QFT. [19] follows the reverse approach starting with $QFT \to QM \to CM$. I am not saying these are the best books to study QFT, which is definitely not true. I am merely pointing out that these books are organized similarly to these notes. Also, check [20, 21] for short notes related to fundamental physics and [7, 8] for relevant mathematics notes.

2 Mathematics

> The book of nature is written in the language of mathematics.
>
> *Galileo Galilei*

[2–8]. Main reference is [2].

2.1 Mathematical preliminaries

2.2 Real analysis

2.3 Complex analysis

Wirtinger derivatives: In what sense are z and $\bar{z}$ independent? In the below, even though $z, \bar{z}$ look independent just like a, b notice that a, b must be real.

$$\begin{cases} z = a + bi \\ z^* = a - bi \end{cases} \iff \begin{cases} a = \frac{1}{2}z + \frac{1}{2}z^* \\ b = \frac{1}{2i}z - \frac{1}{2i}z^* \end{cases}$$

Part I

Classical physics

> If nature were not beautiful it would not be worth knowing, and life would not be worth living.
>
> *Henri Poincaré*

3 Classical mechanics

In classical mechanics, we study point particles. There is only one type of Newtonian particle in classical mechanics, unlike in quantum mechanics. These particles follow the Maxwell–Boltzmann statistics. These particles interact with classical fields like Newtonian gravity, electromagnetism, etc.

3.1 Newtonian formulation

Are Newton's laws of motion just definitions or empirical facts [22]? Recall that Newton's laws say that in inertial frames

1. A body remains at rest, or in motion at a constant speed in a straight line, except insofar as it is acted upon by a force.

2. The net force on a body is equal to the body's instantaneous acceleration multiplied by its instantaneous mass or, equivalently, the rate at which the body's momentum changes with time.

3. If two bodies exert forces on each other, these forces have the same magnitude but opposite directions.

The above traditional versions are bad. Because the 2nd law can be considered either as the definition of $\mathbf{F}(\mathbf{x}(t)) = m\frac{\mathrm{d}^2\mathbf{x}(t)}{\mathrm{d}t^2}$ or the definition of inertial frames. The 1st law is just a corollary of the 2nd when the net force is $\mathbf{0}$. The 3rd one is an empirical fact, which is true if the forces are instantaneous. In classical field theories like electromagnetism, due to the force field not propagating instantaneously, it is not true. The below are better versions;

1. Inertial reference frames exist.

2. Forces (quantification of how much interaction there is) between particles behave like mathematical vectors, and the acceleration of the particle is proportional to the net vector addition of all forces, and mass is defined as the inverse of the proportionality constant.

3. Same as before.

This 1st law is saying that there exists an inertial frame. Once you believe in that, you can do experiments in any random reference frame, and as long as you carefully add fictitious forces like Coriolis force, centrifugal force, etc then Newton's laws will work. This 2nd law is an empirical fact that force behaves like a vector and not like some other thing like scalar or pseudo-vector or spinor, etc. It is also an empirical fact that it is proportional to acceleration and not $\frac{\mathrm{d}^n\mathbf{x}(t)}{\mathrm{d}t^n}$ for some $n > 2$. This is why you only need the positions and velocities of all Newtonian particles for complete knowledge to predict everything.

3.2 Lagrangian formulation

3.3 Hamiltonian formulation

3.4 Hamilton–Jacobi formulation

3.5 Statistical thermodynamics

3.6 Nonlinear dynamics and chaos

3.7 Special relativity

Random history: Einstein's contribution to special relativity is not as significant or remarkable as his contribution to general relativity. In his work on special relativity, he showed that he was a man with enough courage to question fundamental aspects of reality, such as simultaneity and the concept of time, but special relativity is not conceptually deep like general relativity. The mathematics he used for special relativity is very elementary. Lorentz transformations were already known. But people incorrectly interpreted them, using the aether medium before Einstein clarified the meaning of those equations. Initially, Einstein thought that the geometric interpretation introduced by his teacher Minkowski was unnecessarily complicated mathematics introduced into this theory. Only later, he realized the importance of mathematics (especially geometry) when formulating his general relativity. The fact that Einstein was a remarkable genius is only clear from his contributions to general relativity. Unlike in special relativity, where **Lorentz, Poincare, Larmor, FitzGerald and many others** contributed, Einstein ***almost*** single-handedly formulated general relativity (with David Hilbert being the 2nd most important contributor who came up with the correct Einstein's field equations 5 days before Einstein independently but Einstein's paper was published first. But Hilbert rightfully acknowledged Einstein as the main contributor to GR because Einstein previously found the equations with the trace term missing and was aware that he needed to add some term.)

Postulate: Space and time are unified to give the flat Minkowski spacetime, and we can gauge fix the diffeomorphism invariance of special relativity so that the metric will just become $(-1, +1, +1, +1)$[1].

This postulate is better than the original 2 postulates by Einstein as it is easier to generalize to general relativity.

[1] The other signature is considered blasphemy against the laws of physics.

4 Classical field theory

[23–30]
In classical physics, the matter is generally point particles and not fields, but the forces between them are generally classical fields like Newtonian gravity is a *non-relativistic scalar field*, electromagnetism is a *relativistic vector or gauge or spin 1 field*, and Einstein's GR is a spin 2 field. We can also study fermion fields as classical fields with Grassmannian values, but they are not very useful in classical physics because we don't have Fermi–Dirac statistics in classical physics.

4.1 0 : Scalar fields

4.1.1 Newton–Cartan reformulation of Newtonian gravity

Newtonian gravity is the first classical field theory to be discovered, and it is a scalar field theory.

4.1.2 Relativistic scalar field

4.2 $1/2$: Classical spinor or Grassmann fields

4.3 1 : Electromagnetism

4.3.1 Basics

4.3.2 As a $U(1)$ gauge field

4.3.3 In differential forms language

4.3.4 The energy-momentum tensor

4.3.5 Electromagnetic waves

4.3.6 Galilean electromagnetism ($c \to \infty$)

[32]

4.4 1 : Yang-Mills theory

4.5 Spontaneous symmetry breaking

5 General relativity

[33–36]
In principle, this is also a classical field theory, and this section should be a subsection of the previous section. But in some sense, even **classical gravity is secretly already a quantum theory** since even in the classical limit, gravity is dual to some holographic quantum field theory. Apart from that, I really like general relativity (even more than the standard model of particle physics since it has no dimensionless parameters and can be completely guessed by anyone who knows Newtonian gravity and Maxwell's equations without experimental help), so it deserved its own section.

5.1 Formulation

5.1.1 No prior geometry and general covariance

> Mathematics was not sufficiently refined in 1917 to cleave apart the demands for "**no prior geometry**" and for a geometric, coordinate-independent formulation of physics. Einstein described both demands by a single phrase, "general covariance". The "no prior geometry" demand actually fathered general relativity, but by doing so anonymously, disguised as "general covariance", it also fathered half a century of confusion.
>
> *MTW Gravitation* (1973 book)

General covariance or **diffeomorphism invariance** or **reparameterization invariance**: The laws of physics will be invariant under arbitrary differentiable coordinate transformations.

This should have been understood long before general relativity, but this was understood by Einstein while he was developing general relativity and has caused a lot of confusion. Recall Newton's second law

$$\mathbf{F}(\mathbf{x}(t)) = m\frac{\mathrm{d}^2\mathbf{x}(t)}{\mathrm{d}t^2}$$

The above equation is valid for any coordinate system. We generally use Cartesian coordinates because they are the simplest. But if you use spherical coordinates, then the components of the above equation will be

$$\begin{aligned}
F_r &= m(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta) \\
F_\theta &= m(2\dot{r}\dot{\theta} + r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta) \\
F_\phi &= m(2\dot{r}\dot{\phi}\sin\theta + r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)
\end{aligned}$$

Even though the equations look very different, the physics hasn't changed. The reason things became more complicated is because the metric went from $\mathrm{diag}(1, 1, 1)$ to something slightly more complicated. Similarly, we can go to an arbitrary coordinate system with arbitrary metric g_{ab}[2] that is more complicated than spherical coordinates, and Newton's second law is still valid. **We can use a lot of machinery of Riemannian geometry in Newtonian physics** as shown in 4.1.1. But in general, it is **not needed** because, in this case, the background space is not dynamic, and geometry is a priory fixed. We have a flat $3D$ Euclidean space, i.e., scalar curvature is 0 everywhere. So we can always choose (i.e., gauge fixing) the Cartesian coordinates in this case where the metric is simplified to $\mathrm{diag}(1, 1, 1)$. If our space (non-dynamic) instead has some arbitrary curvature, then we can't find nice coordinate systems like Cartesian coordinates, and we should work with nontrivial metrics and use the machinery of Riemannian geometry. **Example**: Think about particles confined to a spherical surface interacting with Newtonian gravity. F_r component will be canceled due to normal confinement forces and $r = const$. This submanifold of $3D$ Euclidean space is a $2D$ **non-dynamic** curved space.

[2] Latin indices are reserved for Euclidean signature. Greek indices are reserved for Minkowski signature.

This logic can be carried over to quantum mechanics, special relativity, quantum field theory etc. All of them **must** have diffeomorphism invariance. In Quantum Mechanics, you probably saw $\vec{\nabla}$ in cartesian, spherical, and cylindrical coordinates. But we can also define an arbitrary coordinate system on $3D$ Euclidean space with some arbitrary metric that will still have 0 scalar curvature everywhere. In special relativity and quantum field theory, we have different coordinates that are not Cartesian, such as the Rindler coordinates[3]. In principle, we can take an arbitrary coordinate system where the metric is very different from $\text{diag}(-1, 1, 1, 1)$. Note that **QFT in curved spacetime $\neq$ Generally covariant QFT**, because in the former, we have an arbitrary non-dynamical spacetime, but in the latter, we have the specific Minkowski spacetime, even though in both cases the coordinate system is arbitrary.

Active and passive transformation: Mathematically they are same. The mathematical equation for your rotation by an angle is the same as if everything else in the universe revolves around you by the same angle. Whether they are physically the same is an **ongoing** philosophical debate. Mach's principle states that the existence of absolute rotation (the distinction of local inertial frames vs. rotating reference frames) is determined by the large-scale distribution of matter in the universe. Though it motivated Einstein to come up with general relativity, it is not exactly known which form of Mach's principle is valid, and theories like Brans–Dicke theory obey a stronger form of Mach's principle than GR.

Diffeomorphism invariance is a gauge symmetry: Recall that only the global part of a gauge symmetry is physical and gives rise to **conserved quantities**. Diffeomorphism invariance is present in any theory. For example, if you consider special relativity and gauge fix the metric so that it becomes $\text{diag}(-1, 1, 1, 1)$, then there is still a global symmetry left corresponding to the Poincaré group. Spacetime translations give 4 momentum conservation. Spacetime rotations give angular momentum conservation (due to spatial rotations) and "conservation of the center of mass" also called the conservation of $\mathbf{N} = t\mathbf{p} - E\mathbf{r}$ (due to Lorentz boosts). General relativity has no new symmetry compared to special relativity, so we don't get any new conserved quantities. Note that if laws have some symmetry, that doesn't mean all solutions to those laws have that symmetry. A notable example is the **FLRW metric**, which doesn't have time translation symmetry due to the initial singularity, and therefore, there is **no notion of energy conservation**. Time-translation symmetry is guaranteed only in spacetimes where the metric is static: that is, where there is a coordinate system in which the metric coefficients contain no time variable.

No prior geometry: This is the main specialty of general relativity compared to previous theories. Because the background is not a priori fixed to Euclidean or Minkowski space and because the background is dynamic, it becomes **absolutely necessary** to use the machinery of Riemannian geometry. Earlier, it was optional.

[3] Recall Unruh effect.

5.1.2 Equivalence principle

5.1.3 Einstein field equations

5.1.4 Einstein–Hilbert action

5.1.5 ADM formalism

5.2 Black holes

5.2.1 Schwarzschild metric

5.2.2 Reissner-Nordström metric

5.2.3 Kerr-Newman metric

[37]

5.2.3.1 Penrose process

5.2.4 The Four Laws

5.2.5 Regular black holes

[38]

5.3 Causal structure

[39]

5.3.1 Singularity theorems

5.3.2 FTL

[40]

5.4 Perturbation theory

5.4.1 GR→NG

The limit to get special relativity is very obvious. The metric just becomes non-dynamical and becomes the Minkowski metric. The limit to get Newtonian Gravity is nontrivial. The following passage from 2.1.4 of [41] explains how even the leading order theory already differs from Newtonian gravity.

"Therefore, general relativity produces the same trajectories at leading order as Newton's theory. This is called the Newtonian approximation of general relativity.

However, let us stress that even at this level of approximation, the two theories differ drastically—in a way that can be tested at the experimental level already. Indeed, in general relativity, the variation of an observer proper time $d\tau$ (Eq. 27) with respect to the proper time of another observer depends explicitly on their different positions in a gravitational potential U . This means that two observers at different locations in the gravitational potential will not agree on the evolution of time. This effect, although minute, can be tested if one has accurate enough clocks. In other words, had we developed atomic clocks with sufficient precision prior to our ability to observe the motions of celestial bodies in the solar system, we could have confirmed the superiority of general relativity over Newton's theory."

5.4.2 Post-Newtonian expansion

5.4.3 Minkowskian and post-Minkowskian approximation

5.4.4 Gravitational waves

5.5 The Cauchy problem

5.6 Cosmology

[42, 43]

5.6.1 de Sitter space

[44]

5.6.1.1 dS-Schwarzschild metric

5.6.2 Anti-de Sitter space

5.6.2.1 AdS-Schwarzschild metric

5.6.3 FLRW metric

5.6.4 The inhomogeneous universe

[42]

5.6.4.1 Newtonian perturbation theory

5.6.4.2 Relativistic perturbation theory

5.6.4.3 Cosmic microwave background

5.6.5 The standard model of cosmology (ΛCDM)

Note: There is a difference between how GR is related to ΛCDM compared to how QFT is related to the standard model of particle physics. GR is already a single theory. But QFT is a framework that can describe ∞ theories. **Horndeski's theory** [47] is more analogous to QFT than GR. Horndeski's theory is more like a framework for classical gravity theories, and GR is just one of them. But there is a **uniqueness** to GR. GR is the **simplest classical gravity theory** and is preferred by the Occam's razor. In QFT, we don't have this kind of uniqueness. Maybe you can argue that the gauge group $U(1) \times SU(2) \times SU(3)$ is somehow unique, but even then, the dimensionless parameters coming from masses, etc, are arbitrary experimental values. ΛCDM itself is **not a theory** but just a single solution to GR that describes the real universe and is dependent on the initial conditions at the initial singularity. ΛCDM is a *course-grained approximation of the universe*. It explains the large-scale structure of the universe, but it is not an exact model that contains the metric at each point in the universe. Other solutions to GR might also exist in the multiverse. It is possible that ΛCDM is a uniquely preferred solution when we consider quantum gravity (maybe just like electron $g-2$, we can precisely calculate the parameters of ΛCDM), but in GR, it is not the case.

5.6.6 Inflation

5.6.7 Modified gravity

[45–47]

5.6.7.1 Horndeski's theory

[47]

5.6.8 Ultimate fate of the universe scenarios

Part II
Quantum physics

6 Quantum mechanics

[48]

6.1 Formulation

6.1.1 Canonical formulation

6.1.2 Path integral formulation

6.2 Rotations and angular momentum

6.3 Perturbation theory

6.4 QM→CM

References: Chapter 14 of [48].

6.5 Axiomatic QM

[49]

6.5.1 Dirac–von Neumann axioms and C*-algebras

6.5.2 Spectral theory

6.5.2.1 Rigged Hilbert spaces

6.6 Quantum information theory

[50]

6.7 Statistical thermodynamics

6.8 Relativistic quantum mechanics (RQM)

[51–54]

6.8.1 0 : Klein–Gordon equation

6.8.2 $1/2$: Dirac, Weyl and Majorana equations

[55]

6.8.3 $1:$ Maxwell equation and Proca equation

6.8.4 $3/2:$ Rarita–Schwinger equation

6.8.5 $j:$ Bargmann–Wigner equation and Joos–Weinberg equation

7 Quantum field theory

7.1 $0:$ Scalar fields

7.2 $1/2:$ Fermion fields

7.3 $1:$ Gauge fields

7.3.1 QED

7.3.2 Yang–Mills theory

7.4 Scattering amplitudes

[56, 57]

7.5 Renormalization

7.5.1 QED

7.5.2 Yang–Mills theory

7.6 Spontaneous symmetry breaking

7.6.1 Abelian Higgs mechanism

7.7 Anomalies

7.8 Solitons

[58]

7.9 Nonrelativistic QFT (NQFT)

[59]

7.10 QFT→NQFT or RQM or QM

References: Chapter 8 of [9], [60] and Chapter 6 of [19].

7.11 The standard model of particle physics

[61–63]
We should now touch some grass and make contact with reality. Using QFT (it's a framework, not a theory), you can make infinite theories, and the standard model is 1 of them that we can fix based on experiments. It needs 25 fundamental dimensionless constants determined by experiments. See the **Note** in 5.6.5.

7.11.1 Electroweak theory

7.11.2 QCD

[64]

7.11.2.1 Confinement

7.12 BSM (Beyond the Standard Model)

[65]

7.12.1 Neutrino oscillations

7.12.2 Dark matter candidates

7.12.3 Baryon asymmetry

[66]

7.12.4 Grand unified theories

7.13 Lattice gauge theory

[67]

7.14 Effective field theory

[68–71]

7.15 QFT in lower dimensions

[72, 73]

7.15.1 0+0

7.15.2 0+1

7.16 The large N limit

7.17 Entanglement

[74, 75]

7.18 Axiomatic quantum field theory

[76]

7.18.1 Problems with nonrigorous QFT

7.18.1.1 Haag's theorem

[77]

7.18.2 Wightman axioms

[78–80]

7.18.3 Constructive QFT

[81–83]

7.18.4 Haag–Kastler algebraic QFT

[84, 85]

A Failed theories

1. Aristotelian physics (384–322 BC)
2. Brahmagupta's gurutvākarṣaṇam qualitative theory of gravity (628)
3. Descartes' vortices theory of gravity (1644)
4. Aether theories before special relativity (1704-1905)
5. Einstein's scalar field theory for gravity (1912)
6. Old semiclassical quantum theory (1900–1925)
7. Relativistic quantum mechanics (1927)
8. Steady-state model (1930s and 40s)
9. Einstein's classical unified theory (1930-1955)
10. Technicolor (1980s)
11. String theory (as an alternative to QCD it failed, as quantum gravity, it still is the most promising candidate)

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